Matrix multiplication can be used in graphics to rotate, shear, scale, or translate an image. Consider the blue triangle shown in the figure with vertices at \((-3, 0), (0, 3),\) and \((3, 0)\). We can form a matrix \(T = \begin{bmatrix} -3 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix}\) to describe this. Note that the columns store the points in the form \((x, y)\). The matrix \(S = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}\) corresponds to a shear. The matrix \(ST = \begin{bmatrix} -3 & 1.5 & 3 \\ 0 & 3 & 0 \end{bmatrix}\) contains the vertices of the red triangle.

(a) Find the image of the blue triangle under the transformations with each of the following matrices and plot the results.

\[
S_1 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad S_3 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}
\]

(b) Draw the figure that has adjacent vertices \((-2, 0), (-1, 3), (0, 2), (3, 2)\) and \((3, 0)\) connected by straightline segments. Perform the same transformations as in part (a) by first constructing a matrix (like \(T\)) and using the matrix product. Draw the transformed figures.
(2) A square matrix with the same number in each diagonal entry, 1 in each entry just above the diagonal, and zero everywhere else is called a Jordan block. For example, \[
\begin{bmatrix}
2 & 1 & 0 \\
0 & 2 & 1 \\
0 & 0 & 2
\end{bmatrix}
\]
is a Jordan block. Consider the Jordan block matrix
\[
A = \begin{bmatrix}
x & 1 \\
0 & x
\end{bmatrix}.
\]

(a) Compute the first few powers of \(A\), namely \(A^2\), \(A^3\), and \(A^4\).

(b) Make a conjecture as to the general power \(A^n\) for integer \(n \geq 2\).

(c) Notice anything calculus like about \(A^n\)?