# September 17 MATH 1113 sec. 52 Fall 2018

#### Algebra of Rational Expressions (JIT 22, 23, 24)

A **rational expression** is a ratio (i.e. a fraction) in which the numerator and denominator are polynomial expressions. Examples include

$$\frac{x^3+4x^2}{x^2-16}$$
 and  $\frac{2a}{a+3}$ .

We will study *rational functions* in the next section. So here we refresh the basic algebra (addition, subtraction, multiplication, division) and simplification involving rational expressions.

It draws on our knowledge of rational numbers and polynomials.

# Example

Evaluate the sum and simplify if possible.

$$\frac{x}{x^2-1} - \frac{2}{x^2+2x-3}$$
  
be need a common denominator.  
We'll try to find the least  
Common denominator (LCD).  
Start by factoring the  
denominators  
our LCD has to have  
 $x^2 - 1 = (x-1)(x+1)$   
 $x^2 + 2x - 3 = (x - 1)(x + 3)$   
 $x^2 + 2x - 3 = (x - 1)(x + 3)$   
 $x - 1, x + 1, x + 3 cs$   
our LCD is  
 $(x - 1)(x + 1)(x + 3)$   
 $(x - 1)(x + 1)(x + 3)$ 

$$\frac{x}{(x-1)(x+1)} - \frac{2}{(x-1)(x+3)}$$

$$\frac{x}{(x-1)(x+1)} \left(\frac{x+3}{x+3}\right) - \frac{2}{(x-1)(x+3)} \left(\frac{x+1}{x+1}\right)$$

$$\frac{x(x+3)}{(x-1)(x+1)(x+3)} - \frac{2(x+1)}{(x-1)(x+3)(x+1)}$$

$$\frac{x^2+3x - (2x+2)}{(x-1)(x+1)(x+3)} : \frac{x^2+3x - 2x - 2}{(x-1)(x+1)(x+3)}$$

$$= \frac{x^2 + x - 2}{(x-1)(x+1)(x+3)}$$

Simplify

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$$= \frac{(x-1)(x+2)}{(x-1)(x+1)(x+3)}$$

$$= \frac{X+2}{(x+1)(X+3)}$$

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### Question

Suppose we wish to evaluate the sum

$$\frac{-8}{x^2-4}+\frac{2}{x-2}.$$

The least common denominator we can use for this operation is

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(a) 
$$(x^2 - 4)(x - 2)$$
  
(b)  $(x^2 + x - 6)$   
(c)  $(x - 2)(x + 2)$ 

(d) 
$$(x-2)^2(x+2)$$

#### Question

d)

$$\frac{-8}{x^2 - 4} + \frac{2}{x - 2} = \frac{-8}{(x \cdot v)(x + 1)} + \frac{2(x + 1)}{(x - 1)(x + 1)}$$
(a)  $\frac{-6}{x^2 - 4}$ 
(b)  $\frac{2x + 10}{x^2 - 4}$ 
(c)  $\frac{-6}{x^2 + x - 6}$ 
(c)  $\frac{-6}{x^2 + x - 6}$ 
(c)  $\frac{2x - 4}{x^2 + x - 6}$ 
(c)  $\frac{-2}{x^2 + x - 6}$ 
(c)  $\frac{2x - 4}{(x - 1)(x + 2)} - \frac{2(x - 1)}{(x - 1)(x + 2)}$ 
(c)  $\frac{2}{x + 2}$ 

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# Simplifying Complex Rational Expressions

A **complex**<sup>1</sup> rational expression is one in which the numerator or denominator (or both) contain a rational expression—i.e. fraction within a fraction. Examples include

$$\frac{\frac{1}{x}+1}{1-\frac{1}{x}} \quad \text{and} \quad \frac{\frac{w}{v}+\frac{v}{w}}{wv}$$

We wish to rewrite such as thing as a rational expression that is no longer complex.

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<sup>&</sup>lt;sup>1</sup>also called *compound* rational expressions

#### Example: Simplify the complex rational expression. We'll use two different approaches. First, we'll simplify numerator and denominator before performing the division.

Work with numerator & Penonitor first  $\frac{1}{x} - \frac{1}{y} = \frac{1}{x} \frac{\partial}{\partial y} - \frac{1}{y} \frac{x}{x} = \frac{y}{xy} - \frac{x}{xy} = \frac{y-x}{xy}$  $\begin{array}{c} L_{\chi^{2}} - \frac{1}{2\gamma} = \frac{1}{\chi^{2}} \frac{y^{2}}{\gamma^{2}} - \frac{1}{2\gamma} \frac{\chi^{2}}{\chi^{2}} = \frac{y^{2}}{\chi^{2}} - \frac{\chi^{2}}{\chi^{2}} \frac{y^{2} - \chi^{2}}{\chi^{2}} \\ \end{array}$ 

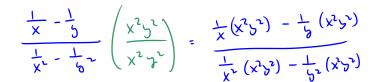
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y-y) ×y (y-×)×5 (y-x)(y+x) Y+X א - ז

(日)

#### Example: Simplify the complex rational expression. Now we do this again by clearing the fractions.

$$\frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x^2} - \frac{1}{y^2}}$$
Look for the LCD of all  
ratios inside the expression.  
Here  $x^2y^2$ . mult by  $\frac{LCO}{LCO}$ 



xy (y-x) xy2 - x<sup>2</sup>y χ۰  $\overline{(y-x)(y+x)}$ t y² y + X ΧŤ

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