September 24 MATH 1113 sec. 51 Fall 2018

Section 4.5: Rational Functions

Plot $f(x) = \dfrac{x^2 - 3x - 4}{x^2 - 1}$.

Determine the domain, and put f into lowest terms.

factor $\quad f(x) = \dfrac{(x-4)(x+1)}{(x-1)(x+1)}$

Domain $\quad \{x \mid x \neq \pm 1\}$

In lowest terms $\quad f(x) = \dfrac{x-4}{x-1}, \; x \neq -1$

$$f(x) = \frac{x^2-3x-4}{x^2-1}$$

Find the equation(s) of any vertical asymptotes.

$f(x) = \dfrac{x-4}{x-1}$, $x \neq -1$

One Vert. asymptote $x = 1$.

$f(x) = \frac{x^2-3x-4}{x^2-1} = \frac{x-4}{x-1}$, $x \neq -1$

Identify any horizontal or oblique asymptote, and identify any points at which the graph crosses.

In lowest terms the degree of the numerator is $n = 1$ and that of the denominator is $m = 1$.

$n = m \Rightarrow$ there is a horizontal asymptote

$y = \frac{1}{1} = 1$ that is, $y = 1$

Does is cross: $f(x) = 1$ $\frac{x-4}{x-1} = 1 \Rightarrow x-4 = x-1$

$\Rightarrow -4 = -1$ false The graph doesn't cross.

$f(x) = \frac{x^2-3x-4}{x^2-1} = \frac{x-4}{x-1}$ for $x \neq -1$

Identify the points of any x and y intercepts.

y-intercept $f(0) = \frac{0-4}{0-1} = 4$ $(0,4)$

x-intercept $f(x) = 0 \Rightarrow 0 = \frac{x-4}{x-1}$

$\Rightarrow x-4 = 0 \Rightarrow x = 4$

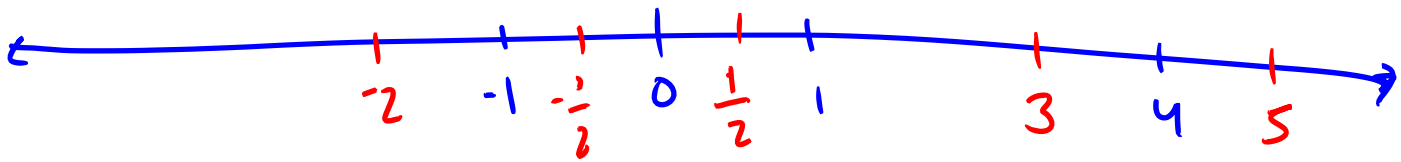
$(4,0)$

$f(x) = \frac{x^2-3x-4}{x^2-1} = \frac{x-4}{x-1}$, $x = -1$

Identify points on the graphs—in particular points between intercepts and vertical asymptotes.

Hole @ -1 $\quad \frac{-1-4}{-1-1} = \frac{-5}{-2} = \frac{5}{2}$

Hole is @ $(-1, \frac{5}{2})$



Test points in red

$f(x) = \frac{x^2-3x-4}{x^2-1} = \frac{x-4}{x-1}$, $x \neq -1$

Identify points on the graphs—in particular points between intercepts and vertical asymptotes.

$f(-2) = 2$, $f(-\frac{1}{2}) = 3$, $f(\frac{1}{2}) = 7$

$f(3) = -\frac{1}{2}$ and $f(5) = \frac{1}{4}$

$$f(x) = \frac{x^2-3x-4}{x^2-1}$$

Interval	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,4)$	$(4,\infty)$	
test pt c	-2	$-\frac{1}{2}$	$\frac{1}{2}$	3	5	
$f(c)$	2	3	7	$-\frac{1}{2}$	$\frac{1}{4}$	
sign	$+$	$+$	$+$	$-$	$+$	

$$f(x) = \frac{x^2-3x-4}{x^2-1}$$