September 24 MATH 1113 sec. 52 Fall 2018

Section 4.5: Rational Functions

We can obtain a graph of a rational function in steps and by identifying key features. These include:

- the domain of the function,
- putting it in lowest terms,
- finding any vertical asymptotes
- finding a horizontal or an oblique asymptote if one exists (determine if the graph crosses)
- find the y-intercept if 0 is in the domain (i.e. find $f(0)$)
- find any x-intercepts (i.e. solve $p(x) = 0$)
- identify behavior near asymptotes (plot at least one point between each intercept and vert. asymptote)

Plot $f(x) = \frac{x+4}{x^2-3}$ [1]

Determine the domain, and put f into lowest terms.

$$f(x) = \frac{x+4}{(x-\sqrt{3})(x+\sqrt{3})}$$

The denominator is zero if $x=\sqrt{3}$ or $x=-\sqrt{3}$.

These numbers are <u>not</u> in the domain.

The domain is $\{x \mid x \neq \pm\sqrt{3}\}$.

In interval notation $(-\infty, -\sqrt{3}) \cup (-\sqrt{3}, \sqrt{3}) \cup (\sqrt{3}, \infty)$.

f is in lowest terms.

[1] We'll plot on the graph a few slides down.

$$f(x) = \frac{x+4}{x^2-3} = \frac{x+4}{(x-\sqrt{3})(x+\sqrt{3})}$$

Find the equation(s) of any vertical asymptotes.

There are two vertical asymptotes

$x = \sqrt{3}$ and $x = -\sqrt{3}$.

$f(x) = \frac{x+4}{x^2-3}$

Identify any horizontal or oblique asymptote, and identify any points at which the graph crosses.

The degree n of the numerator is 1.

The degree m of the denominator is 2.

$n < m$ There is a horizontal asymptote

$y = 0$.

Does it cross? $f(x) = y$ (when $y=0$)

$$\frac{x+4}{x^2-3} = 0 \Rightarrow x+4=0 \Rightarrow x=-4$$

It crosses @ $(-4, 0)$.

$f(x) = \frac{x+4}{x^2-3}$

Identify the points of any x and y intercepts.

y-intercept set $x = 0$

$$f(0) = \frac{0+4}{0^2-3} = \frac{-4}{3}$$

$\left(0, \frac{-4}{3}\right)$

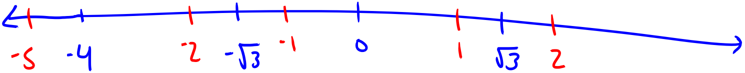
x-intercept set $f(x)$ equal to zero.

we found it at $(-4, 0)$.

$f(x) = \frac{x+4}{x^2-3}$

Identify points on the graphs—in particular points between intercepts and vertical asymptotes.

Let's split up the real line by the interesting x-values we have



Will use test values in red.

$f(x) = \frac{x+4}{x^2-3}$

Identify points on the graphs—in particular points between intercepts and vertical asymptotes.

$f(-5) = \frac{-1}{22}$ $\quad f(-2) = 2$, $\quad f(-1) = \frac{-3}{2}$

$f(1) = \frac{-5}{2}$ $\quad f(2) = 6$

$f(x) = \frac{x+4}{x^2-3}$

Interval	$(-\infty, -4)$	$(-4, -\sqrt{3})$	$(-\sqrt{3}, 0)$	$(0, \sqrt{3})$	$(\sqrt{3}, \infty)$	
test pt c	-5	-2	-1	1	2	
$f(c)$	$\frac{-1}{22}$	2	$\frac{-3}{2}$	$\frac{-5}{2}$	6	
sign	$-$	$+$	$-$	$-$	$+$	

$f(x) = \frac{x+4}{x^2-3}$