September 26 MATH 1113 sec. 52 Fall 2018

Section 2.1: Average Rate of Change & Difference Quotients

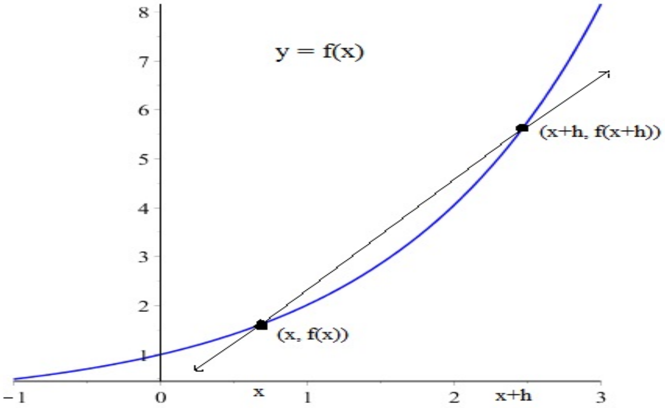


Figure: Consider points $(x, f(x))$ and $(x+h, f(x+h))$ on the graph of f and the straight line through them. This line is called a **secant** line.

Average Rate of Change: Difference Quotients

Let f be defined at x and $x+h$ for nonzero number h. The line passing through $(x, f(x))$ and $(x+h, f(x+h))$ is called a **secant** line. Determine its slope.

Let $(x, f(x)) = (x_1, y_1)$

$(x+h, f(x+h)) = (x_2, y_2)$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{x+h-x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

The result is called a **difference quotient**. It is the **average rate of change** of f on the interval with end points x and $x+h$.

Example

For $h \neq 0$, construct and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$ for $f(x) = \frac{1}{x}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h}\left(\frac{x(x+h)}{x(x+h)}\right)$$

$$= \frac{x-(x+h)}{hx(x+h)} = \frac{x-x-h}{hx(x+h)}$$

$$= \frac{-h}{hx(x+h)} = \frac{-1}{x(x+h)}$$

Question

The difference quotient $\frac{f(x+h)-f(x)}{h}$ for $f(x) = 3x^2 - x$ is

(a) $6x + 3h - 1$

(b) $3h + 1$

(c) $\dfrac{3h^2 - 2x + h}{h}$

(d) $6x - 1$

(e) none of the above is the correct answer

Section 2.2: Piecewise Defined Functions

We wish to consider functions that are defined by different rules over different parts of the domain. These are called piecewise defined functions. An example is

$$f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 2, & 0 \le x < 1 \\ \frac{1}{x}, & x \ge 1 \end{cases}$$

We can plot this if we know how to graph

$y = x^2 - 1$, $y = 2$ and $y = \frac{1}{x}$

Example Evaluating Piecewise Defined Functions

Let $f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 2, & 0 \le x < 1 \\ \frac{1}{x}, & x \ge 1 \end{cases}$

Evaluate

(a) $f(4) = \frac{1}{4}$ $4 \geq 1$

(b) $f(-\pi) = (-\pi)^2 - 1 = \pi^2 - 1$ $-\pi < 0$

(c) $f(0) = 2$ $0 \le 0 < 1$

(d) $f\left(\frac{1}{3}\right) = 2$ $0 \le \frac{1}{3} < 1$

Question

Evaluate $g(2)$ where

$$g(x) = \begin{cases} \frac{x+1}{x-2}, & x \leq -3 \\ x^3 - 2x, & -3 < x < 1 \\ \frac{x}{2} - 3, & x \geq 1 \end{cases}$$

(a) $g(2)$ is undefined because of division by zero

(b) $g(2) = 4$

(c) $g(2) = -2$

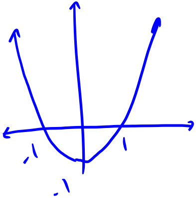
(d) $g(2)$ is undefined because 2 is not in the domain

Plotting Piecewise Defined Functions

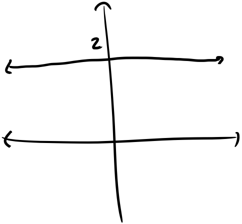
If we know how to plot the different pieces of a piecewise defined function, then we can sketch its plot. As an example, let's plot

$$f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 2, & 0 \le x < 1 \\ \frac{1}{x}, & x \ge 1 \end{cases}$$

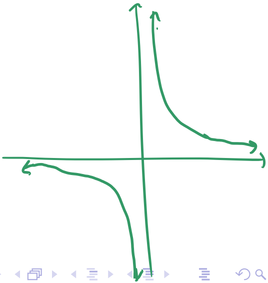
$y = x^2 - 1$



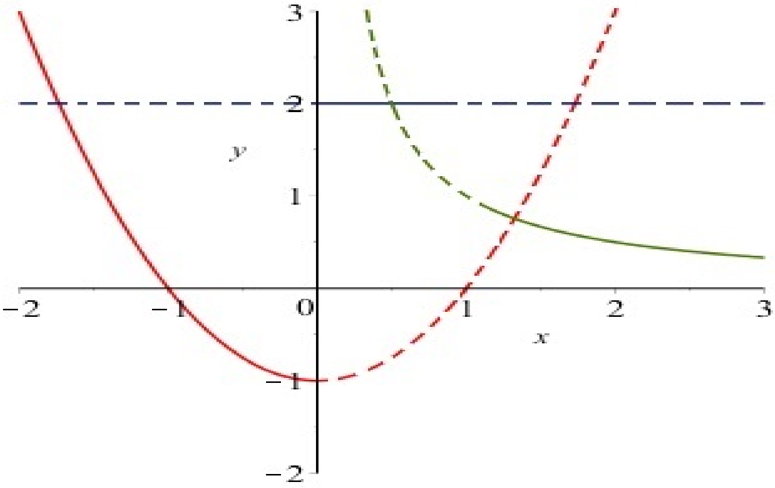
$y = 2$



$y = \frac{1}{x}$

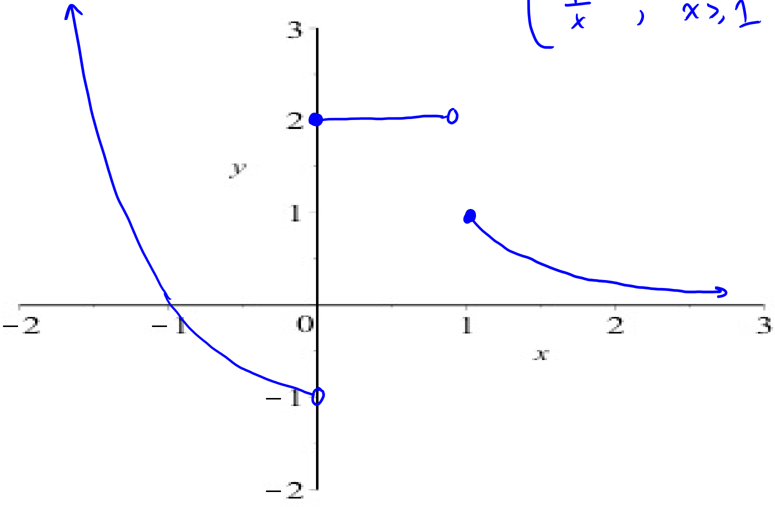


Plotting Example



Plotting Example

$$f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 2, & 0 \le x < 1 \\ \frac{1}{x}, & x \geq 1 \end{cases}$$



Piecewise Defined Functions

Let $f(x) = \begin{cases} \frac{x}{x-2}, & x < 2 \\ 1, & x = 2, \\ x^2, & x > 2 \end{cases}$. Suppose that $h > 0$, and evaluate

(a) $f(2) = 1$

(b) $f(2+h) = (2+h)^2 = 4+4h+h^2$

Since $h>0$ $2+h>2$

(c) $f(2-h) = \dfrac{2-h}{2-h-2}$

$= \dfrac{2-h}{-h} = \dfrac{h-2}{h}$

$2-h < 2$

Question

Let $H(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ 0, & x = 1 \end{cases}$ Then for $h \neq 0$, $\dfrac{H(1+h) - H(1)}{h} =$

(a) $\dfrac{h^2 + 2h}{h+1}$

(b) $\dfrac{h+2}{h+1}$

(c) $\dfrac{2+h}{h}$

(d) is undefined.