September 7 MATH 1113 sec. 51 Fall 2018

Section 3.2 & 3.3: Quadratic Functions and Quadratic Equations

To solve quadratic equations or to plot the graph of a quadratic function, it is useful to express a quadratic in a new form sometimes called **vertex form.**

If $\quad f(x) = a(x-h)^2 + k,$

then the graph of f has vertex at the point (h,k) and is symmetric about the vertical line $x = h$. Moreover, if f has a real zero x_0, then

$$x_0 = h + \sqrt{-\frac{k}{a}} \quad \text{or} \quad x_0 = h - \sqrt{-\frac{k}{a}}.$$

Completing the Square

We determined that if

$$f(x) = ax^2 + bx + c = a(x - h)^2 + k$$

then

$$h = -\frac{b}{2a} \quad \text{and} \quad k = \frac{4ca - b^2}{4a}$$

Completing the Square

Let $f(x) = 3x^2 - 12x + 7$. Complete the square to write f in the form $f(x) = a(x-h)^2 + k$.

We can follow a procedure.

① Separate the x, x^2 terms and factor out a

$$3x^2 - 12x + 7 = (3x^2 - 12x) + 7$$
$$= 3(x^2 - 4x) + 7$$

② Use $(x-h)^2 = x^2 - 2hx + h^2$, then add and subtract

$$h^2 = \left(\frac{-b}{2a}\right)^2$$

$$3x^2 - 12x + 7 = 3\left(x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2\right) + 7$$

$$= 3(x^2 - 4x + 4) - 3 \cdot 4 + 7$$

3 Then $x^2 - \frac{b}{a}x + \frac{b^2}{4a^2}$ is $(x-h)^2$

$$3x^2 - 12x + 7 = 3(x-2)^2 - 12 + 7$$

$$= 3(x-2)^2 - 5$$

So $f(x) = 3(x-2)^2 - 5$

Note $a = 3$, $b = -12$, $c = 7$

$h = 2$ and $k = -5$

$$h = \frac{-b}{2a} = \frac{-(-12)}{2 \cdot 3} = \frac{12}{6} = 2$$

$$k = \frac{4ac - b^2}{4a} = \frac{4 \cdot 7 \cdot 3 - 12^2}{4 \cdot 3}$$

$$= \frac{12(7 - 12)}{12} = 7 - 12 = -5$$

Example

Find the zeros of the function $f(x) = 3x^2 - 12x + 7$, and find its minimum value.

$f(x) = 3(x-2)^2 - 5$

The vertex is $(2, -5)$. The minimum value of f is -5.

To find zeros, solve $f(x) = 0$

$3(x-2)^2 - 5 = 0$

$3(x-2)^2 = 5$

$$(x-2)^2 = \frac{5}{3}$$

By the principle of square roots

$$x-2 = \sqrt{\frac{5}{3}} \quad \text{or} \quad x-2 = -\sqrt{\frac{5}{3}}$$

Calling them x_0 and x_1

$$x_0 = 2+\sqrt{\frac{5}{3}} \quad \text{and} \quad x_1 = 2-\sqrt{\frac{5}{3}}$$

The x-intercepts are

$$\left(2+\sqrt{\frac{5}{3}},\ 0\right) \quad \text{and} \quad \left(2-\sqrt{\frac{5}{3}},\ 0\right)$$

Question

The quadratic function f has its vertex at $(-2, 4)$ and is open downward. Which of the following could be the function f?

$f(x) = a(x+2)^2 + 4$

a negative

(a) $f(x) = (x-2)^2 + 4$

(b) $f(x) = -3(x+2)^2 - 4$

(c) $f(x) = 4 - (x+2)^2$ $= -(x+2)^2 + 4$

(d) $f(x) = -2(x-4)^2$

(e) None of the above functions has the right properties.

Discriminant

Given $ax^2 + bx + c$, the **discriminant** of the quadratic is the number

$$b^2 - 4ac.$$

Theorem: The quadratic equation $ax^2 + bx + c = 0$ has

(a) no real solutions if $b^2 - 4ac < 0$

(b) one real solution if $b^2 - 4ac = 0$, and

(c) two distinct real solutions if $b^2 - 4ac > 0$.

Definition: A quadratic polynomial is called **irreducible** if it's discriminant is negative.

Question

The discriminant of $3x^2 - 12x + 7$ is

(a) 60

$b^2 - 4ac = (-12)^2 - 4 \cdot 3 \cdot 7$

(b) -228

$= 12(12 - 7) = 12 \cdot 5 = 60$

(c) -60

(d) 228

Question

Suppose $f(x) = (x-h)^2 + k$ and $k > 0$ (i.e. k is positive). Which of the following must be true about the x-intercepts of f?

(a) The point (h, k) is the only x-intercept.

(b) f has two different x-intercepts.

vertex above x-axis

open upwards

(c) f doesn't have any x-intercepts because the discriminant is $-4k$ which is negative.

(d) Nothing can be said about x-intercepts without knowing the sign of h.