September 7 MATH 1113 sec. 52 Fall 2018

Section 3.2 & 3.3: Quadratic Functions and Quadratic Equations

To solve quadratic equations or to plot the graph of a quadratic function, it is useful to express a quadratic in a new form sometimes called **vertex form.**

If $\quad f(x) = a(x-h)^2 + k,$

then the graph of f has vertex at the point (h, k) and is symmetric about the vertical line $x = h$. Moreover, if f has a real zero x_0, then

$$x_0 = h + \sqrt{-\frac{k}{a}} \quad \text{or} \quad x_0 = h - \sqrt{-\frac{k}{a}}.$$

Completing the Square

If $f(x) = a(x-h)^2 + k$ has a real zero x_0, show that

$$x_0 = h + \sqrt{-\frac{k}{a}} \quad \text{or} \quad x_0 = h - \sqrt{-\frac{k}{a}}.$$

If x_0 is a zero of f, then $f(x_0) = 0$.

If $a(x_0 - h)^2 + k = 0 \Rightarrow a(x_0-h)^2 = -k$

$(x_0 - h)^2 = \frac{-k}{a}$ by the principle of square roots

$x_0 - h = \sqrt{\frac{-k}{a}}$ or $x_0 - h = -\sqrt{\frac{-k}{a}}$ $\Rightarrow$ $x_0 = h + \sqrt{\frac{-k}{a}}$ or $x_0 = h - \sqrt{\frac{-k}{a}}$

Completing the Square

The coefficients of the two forms are related. If

$$f(x) = ax^2 + bx + c = a(x-h)^2 + k$$

then

$$h = -\frac{b}{2a} \quad \text{and} \quad k = \frac{4ca - b^2}{4a}$$

Completing the Square

Let $f(x) = 3x^2 - 12x + 7$. Complete the square to write f in the form $f(x) = a(x-h)^2 + k$.

We'll follow a process

① Isolate the x, x^2 terms and factor out a

$3x^2 - 12x + 7 = (3x^2 - 12x) + 7$

$= 3(x^2 - 4x) + 7$

$-4 = -2h$

$(-h)^2 = \left(\frac{-4}{2}\right)^2$

② Since $(x-h)^2 = x^2 - 2hx + h^2$ we need to add and subtract $h^2 = \left(\frac{-b}{2a}\right)^2$

$3x^2 - 12x + 7 = 3\left(x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2\right) + 7$

$$= 3(x^2 - 4x + 4 - 4) + 7$$

$$= 3(x^2 - 4x + 4) - 3 \cdot 4 + 7$$

$$= 3(x-2)^2 - 12 + 7$$

$$= 3(x-2)^2 - 5$$

So vertex form is $f(x) = 3(x-2)^2 - 5$

Note

$a = 3$

$b = -12$

$c = 7$

$$h = \frac{-b}{2a}$$

$$= \frac{-(-12)}{2 \cdot 3} = \frac{12}{6} = 2$$

$$k = \frac{4ac - b^2}{4a} = \frac{4 \cdot 3 \cdot 7 - (-12)^2}{4 \cdot 3}$$

$$= \frac{12 \cdot 7 - 12 \cdot 12}{12} = \frac{12(7-12)}{12} = 7 - 12$$

$$= -5$$

Example

Find the zeros of the function $f(x) = 3x^2 - 12x + 7$, and find its minimum value.

$f(x) = 3(x-2)^2 - 5$ The vertex is @ $(2,-5)$.

The minimum value of f is -5.

If f has zeros then $f(x) = 0$.

solving for x

$$3(x-2)^2 - 5 = 0$$

$$3(x-2)^2 = 5$$

$$(x-2)^2 = \frac{5}{3}$$

By the principle of square roots

$$x-2 = \sqrt{\frac{5}{3}} \quad \text{or} \quad x-2 = -\sqrt{\frac{5}{3}}$$

Calling the solutions x_0 and x_1

$$x_0 = 2 + \sqrt{\frac{5}{3}} \quad \text{and} \quad x_1 = 2 - \sqrt{\frac{5}{3}}$$

Question

The quadratic function f has its vertex at $(-2, 4)$ and is open downward. Which of the following could be the function f?

$f(x) = a(x+2)^2 + 4$

a is negative

(a) $f(x) = (x-2)^2 + 4$

(b) $f(x) = -3(x+2)^2 - 4$

(c) $f(x) = 4 - (x+2)^2 = -(x+2)^2 + 4$

(d) $f(x) = -2(x-4)^2$

(e) None of the above functions has the right properties.