# September 7 MATH 1113 sec. 52 Fall 2018

#### Section 3.2 & 3.3: Quadratic Functions and Quadratic Equations

To solve quadratic equations or to plot the graph of a quadratic function, it is useful to express a quadratic in a new form sometimes called **vertex form**.

If 
$$f(x) = a(x-h)^2 + k$$
,

then the graph of *f* has vertex at the point (h, k) and is symmetric about the vertical line x = h. Moreover, if *f* has a real zero  $x_0$ , then

$$x_0 = h + \sqrt{-\frac{k}{a}}$$
 or  $x_0 = h - \sqrt{-\frac{k}{a}}$ .

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## Completing the Square

If  $f(x) = a(x - h)^2 + k$  has a real zero  $x_0$ , show that

$$x_0 = h + \sqrt{-\frac{k}{a}}$$
 or  $x_0 = h - \sqrt{-\frac{k}{a}}$ .

If xo is a zero of f, then f(xo)=0.

If 
$$A(x_{0}-h)^{2}+k=0 \Rightarrow A(x_{0}-h)^{2}=-k$$
  
 $(x_{0}-h)^{2}=\frac{-k}{a}$  by the principle of square roots  
 $x_{0}-h=\sqrt{-\frac{k}{a}} \Rightarrow x_{0}=h+\sqrt{-\frac{k}{a}}$   
or  $x_{0}-h=-\sqrt{-\frac{k}{a}} \Rightarrow x_{0}=h-\sqrt{-\frac{k}{a}}$ 

### Completing the Square

The coefficients of the two forms are related. If

$$f(x) = ax^2 + bx + c = a(x - h)^2 + k$$

then

$$h = -\frac{b}{2a}$$
 and  $k = \frac{4ca - b^2}{4a}$ 

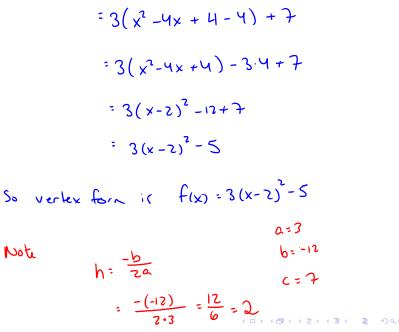
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## Completing the Square

Let  $f(x) = 3x^2 - 12x + 7$ . Complete the square to write f in the form  $f(x) = a(x-h)^2 + k$ . Weill follow a process O Isolate the x, x2 terms and factor out a  $3x^{2} - 12x + 7 = (3x^{2} - 12x) + 7$ -4 = -2h(-h)2 = (-4) = 3(x<sup>2</sup> - 4x) + 7 (3) Since  $(x-h)^2 = x^2 - 2hx + h^2$  we need to add and subtract  $h^2 = \left(\frac{-b}{2a}\right)^2$  $3x^{2} - 12x + 7 = 3(x^{2} - 4x + (\frac{-4}{2})^{2} - (\frac{-4}{2})^{2}) + 7$ 



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$$k = \frac{4ac - b^2}{4a} = \frac{4.3.7 - (-12)^2}{4.3}$$

$$\frac{-12\cdot 7 - 12\cdot 12}{12} = \frac{-12}{12} = \frac{-12}{12} = \frac{-12}{12}$$

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# Example

Find the zeros of the function  $f(x) = 3x^2 - 12x + 7$ , and find its minimum value.

f(x)= 3(x-z) = 5 The vertex is @ (z,-5). The minimum value of fis -5. If I has zeros than fix=0. Solving for X  $3(x-2)^2 - 5 = 0$  $3(x-2)^2 = 5$ 

$$(x-2)^{2} = \frac{5}{3}$$
By the principle of square roots  

$$x-2 = \sqrt{\frac{5}{3}} \text{ or } x-2 = -\sqrt{\frac{5}{3}}$$
Celling the solutions  $x_{0}$  and  $x_{1}$   

$$x_{0} = 2 + \sqrt{\frac{5}{3}} \text{ ond } x_{1} = 2 - \sqrt{\frac{5}{3}}$$

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## Question

The quadratic function f has its vertex at (-2, 4) and is open downward. Which of the following could be the function f?

(a) 
$$f(x) = (x-2)^2 + 4$$
  
(b)  $f(x) = -3(x+2)^2 - 4$   
(c)  $f(x) = 4 - (x+2)^2 = -(x+2)^2 + 4$   
(d)  $f(x) = -2(x-4)^2$ 

(e) None of the above functions has the right properties.

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