

September 9 Math 2306 sec 54 Fall 2015

Section 3.1 (1.3, and a peek at 3.2) Applications

The equation

$$\frac{dP}{dt} = kP(M - P), \quad k, M > 0$$

is called a **logistic growth equation**.

Solve this equation and show that for any $P(0) = P_0 > 0$, $P \rightarrow M$ as $t \rightarrow \infty$.

We separated the variables and used the partial fraction decomposition $\frac{1}{P(M-P)} = \frac{1}{M} \left(\frac{1}{P} + \frac{1}{M-P} \right)$ to conclude that

$$\ln |P| - \ln |M - P| = Mkt + C$$

$$\ln \left| \frac{P}{M-P} \right| = Mkt + C$$

$$e^{\ln \left| \frac{P}{M-P} \right|} = e^{Mkt + C}$$

$$\left| \frac{P}{M-P} \right| = e^C e^{Mkt}$$

$$\frac{P}{M-P} = A e^{Mkt} \quad \text{where } A = \pm e^C$$

$$P = A e^{Mkt} (M-P) \Rightarrow P(1 + A e^{Mkt}) = A M e^{Mkt}$$

$$P = \frac{A M e^{Mkt}}{1 + A e^{Mkt}} \quad P(0) = P_0$$

$$P_0 = \frac{AM e^0}{1 + Ae^0} = P_0 \Rightarrow \frac{AM}{1 + A} = P_0$$

$$AM = P_0(1+A) \Rightarrow A(M - P_0) = P_0$$

$$A = \frac{P_0}{M - P_0}$$

$$P = \frac{\frac{P_0}{M - P_0} M e^{mkt}}{1 + \frac{P_0}{M - P_0} e^{mkt}} \cdot \frac{M - P_0}{M - P_0}$$

$$P = \frac{P_0 M e^{mkt}}{M - P_0 + P_0 e^{mkt}}$$

Take $t \rightarrow \infty$

$$\lim_{t \rightarrow \infty} P = \lim_{t \rightarrow \infty}$$

$$\frac{P_0 M e^{mkt}}{M - P_0 + P_0 e^{mkt}} = \frac{\infty}{\infty}$$

Use
L'Hopital's
rule

$$= \lim_{t \rightarrow \infty}$$

$$\frac{P_0 M (mkt) e^{mkt}}{P_0 (mkt) e^{mkt}}$$

$$= \lim_{t \rightarrow \infty}$$

$$M = M$$

Original Eqn

$$\frac{dP}{dt} = k P (M - P)$$

If $P > M$, $\frac{dP}{dt} < 0$ P decreases toward M

M

If $P < M$ $\frac{dP}{dt} > 0$ P increases toward M .