

There are some suggested tests here. The convergence claims are correct, but you may come up with alternative testing strategies.

Developing your intuition: For each of the following series, guess if it diverges, converges conditionally or converges absolutely. Keep in mind that you must answer two separate questions: 1. Does the series converge? and 2. Does the series converge absolutely? Name the test(s) you would use to answer each of these questions. Usually you are required to give a detailed solution, but for this worksheet, just briefly describe your overall strategy.

1. $\sum_{n=1}^{\infty} \frac{(-1)^n (n + \frac{1}{2})}{n - \frac{1}{2}}$

Diverges
Divergence test

6. $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^{3/2}}$

Converges absolutely
Direct comparison to $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

2. $\sum_{n=1}^{\infty} \frac{(-1)^n n}{e^n}$

Converges Absolutely
Ratio test

7. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3 + n}$

Converges absolutely
Direct comparison to $\sum \frac{1}{n^3}$
can be used on $\sum \left| \frac{(-1)^{n+1}}{n^3 + n} \right|$

3. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

Converges absolutely
Ratio test

8. $\sum_{n=2}^{\infty} \frac{(-1)^n \arctan n}{\sqrt{n}}$

Converges conditionally
Alt. series test and
Direct or limit comparison $\sum \frac{1}{n^{1/2}}$

4. $\sum_{n=1}^{\infty} \frac{(\sin n) 2^n}{n!}$

Converges absolutely
Ratio test

9. $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n^2}$

Converges absolutely.
Can compare $\sum \left| \frac{(-1)^n \ln n}{n^2} \right|$
to $\sum \frac{1}{n^{3/2}}$ if you show $\ln n < \sqrt{n}$

10. $\sum_{n=2}^{\infty} \frac{(-1)^n n}{(\ln n)^2}$

Diverges. Divergence test or $\sum \left| \frac{(-1)^n n}{(\ln n)^2} \right|$

5. $\sum_{n=2}^{\infty} \frac{(-1)^n (n^3 + 1)}{n^4 + n - 4}$

Converges conditionally
Alt. series test + limit comparison to $\sum \frac{1}{n}$

11.
$$\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n^7 + n}}{\sqrt{n^9 + n^5}}$$

Converges conditionally
 Alt. series test +
 limit comparison to $\sum \frac{1}{n}$

12.
$$\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n^7 + n}}{\sqrt{n^{10} + n^5}}$$

Converges absolutely
 Alt. series + limit
 comparison to $\sum \frac{1}{n^{3/2}}$

13.
$$\sum_{n=1}^{\infty} \frac{(-1)^n 10n^2}{n^4 + 1}$$

Converges absolutely
 Alt. series test +
 limit comparison to
 $\sum \frac{1}{n^2}$

14.
$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

Diverges by Integral
 test

15.
$$\sum_{n=1}^{\infty} \frac{n(-2)^n}{n!}$$

Converges
 absolutely
 Ratio test

16.
$$\sum_{n=1}^{\infty} \frac{2 - 5^n}{11^{n-1}(-1)^n}$$

Converges absolutely.
 This is the sum of two
 convergent geometric series
 with $|r_1| = \frac{1}{11}$, $|r_2| = \frac{5}{11}$

17.
$$\sum_{n=1}^{\infty} \sqrt{n} 2^{n+1}$$

Diverges by Divergence
 test

18.
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{4n^5 + n^4 - 1}}$$

Converges absolutely
 Alt. series test + limit
 comparison to $\sum \frac{1}{n^{5/2}}$

19.
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^n n!}{1 \cdot 3 \cdot 5 \cdot 7 \dots (2n-1)}$$

Diverges, ratio test

20.
$$\sum_{n=1}^{\infty} \frac{(-1)^n \sin(n^3)}{2^n}$$

Converges absolutely
 Compar $\sum \frac{|(-1)^n \sin(n^3)|}{2^n}$ to $\sum \frac{1}{2^n}$

21.
$$\sum_{n=1}^{\infty} \frac{(-1)^n n!}{e^{n^2}}$$

Converges
 absolutely
 Ratio test