Snell's Law

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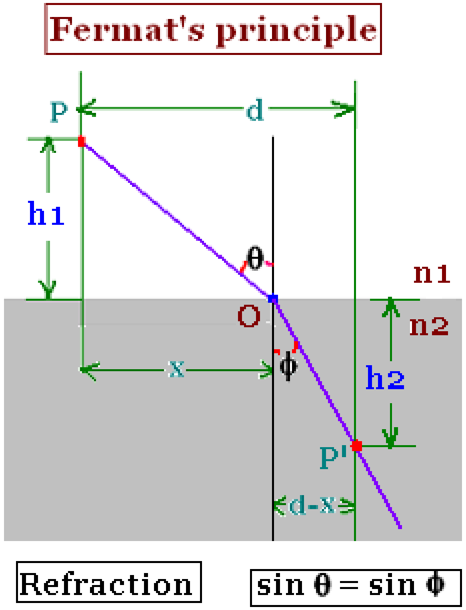
Introduction

- What is Fermat's Principle?
- What is Snell's Law?
- How can these be used?
- How do geometry, distance, and derivations apply to these concepts?

Fermat's Principle

- Discovered by Pierre de Fermat in1658
- Describes light and the path it takes
  - A beam of light will always follow the path which makes its travelling time shortest



Fermat's Principle

Light always moves at a steady speed while it stays in the same medium. For example, in air, light along at $\mathbf{2.998 \times 10^8}$ **meters per second**, but when travelling through water, its speed is a mere $\mathbf{2.254 \times 10^8 ms{-}1.}$

Can you explain why one consequence of Fermat's Principle is that light travelling through just one medium must always travel in a straight line?

- Constant speed
- Must travel in path that takes the least amount of time

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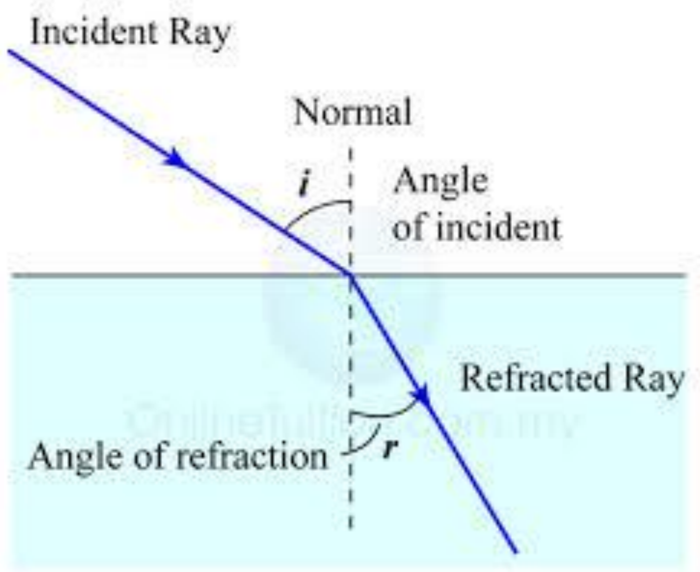
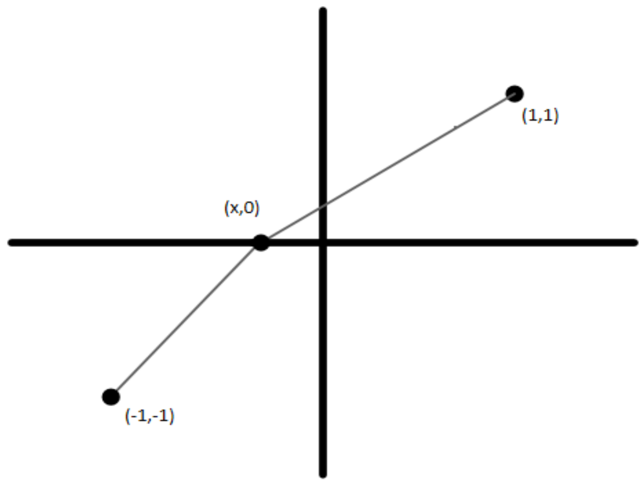
Suppose that the x-axis runs along the surface of a pool of water and that a beam of light is shone from the point A at (1, 1) to the point B at (−1, −1). Draw a picture of the path the beam takes. If the beam of light is to obey Fermat's Principle, find the coordinates of the point C where it must cross from the air into the water.



Distance Formula

- Using Fermat's Principle that light must always travel in the path that takes the least amount of time, we can use the formula $D = RT$ and rewrite it as $T = \frac{D}{R}$.

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$



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- After plugging in the distances into the time equation we get

$$T_{AC} = \frac{1}{R_1}\sqrt{(1-x)^2+1}$$

$$T_{BC} = \frac{1}{R_2}\sqrt{(-1-x)^2+1}$$

Derivations

- Plug in the speed of light in air and water into the equation and minimize the equation because of Fermat's principle and get:

$$T'_{AC} = \frac{-1+x}{(2.998\times10^8)(\sqrt{2-2x-x^2})} \;\&\; T'_{BC} = \frac{1+x}{(2.254\times10^8)(\sqrt{2+2x+x^2})}$$

- Those are just pieces however. To find the total time, we have to add them together.

- After we add the two expressions together, we have to set it equal to zero to find the x-value of the point of incidence

$$0 = \frac{\left(\sqrt{2+2x+x^2}\right)\left(2.254\times10^8\right)(-1+x)+(1+x)\left(2.998\times10^8\right)\left(\sqrt{2-2x+x^2}\right)}{\left(\sqrt{2+2x+x^2}\right)\left(2.254\times10^8\right)\left(2.998\times10^8\right)\left(\sqrt{2-2x+x^2}\right.}$$

$$x = -0.268$$

- The point of incidence is (-0.268,0)

More General Form

- With all of these calculations done, we wanted to find an equation which anyone could solve for the point of incidence just by plugging in the values of the speed of light in the certain mediums. $(C_1 \& C_2)$
- We go through the same derivative taking process and come out with an arbitrary equation of:

$$0 = \frac{(-C_2 + C_2X)\left(\sqrt{2 + 2x + x^2}\right) + (C_1 + C_1X)(\sqrt{2 - 2x + x^2})}{C_2C_1\left(\sqrt{2 + 2x + x^2}\right)(\sqrt{2 - 2x + x^2})}$$

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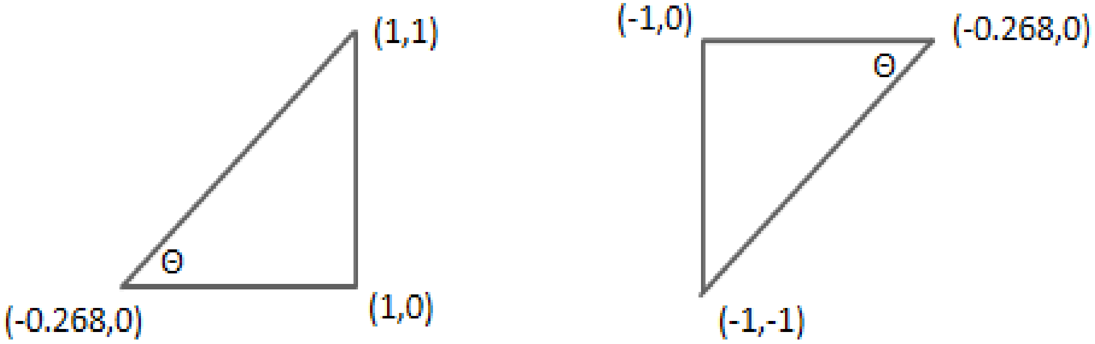
Geometry

- Using the information we have from the two triangles that were formed by the path of the light, we can find out the angles of that certain path of light and make some general statements about the angle and the speeds of the light.



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Generalizations

Triangle 1

$$\tan^{-1} \tan \Theta = \frac{1}{1.268}$$
$$\Theta = 38.261^0$$

Triangle 2

$$\tan^{-1} \tan \Theta = \frac{1}{0.732}$$
$$\Theta = 53.796^0$$

- Using the two triangle above, we can make some generalized statements that must be true.

If $C1>C2$, then $\Theta1<\Theta2$

If $C1<C2$, then $\Theta1>\Theta2$

If $C1=C2$, then $\Theta1=\Theta2$

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Snell's Law

- Using geometry and trig functions, we can see the following in the equations:

$$\frac{x - x_1}{\sqrt{(x - x_1)^2 + {y_1}^2}} = sin\Theta_1$$

$$\frac{x - x_2}{\sqrt{(x_2 - x)^2 + {y_2}^2}} = sin\Theta_2$$

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Snell's Law

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- By using those identities and plugging them back into the total time equation that we used earlier, in addition to introducing the speed of light factors, we get

$$\frac{C_o}{V_1} sin\Theta_1 = \frac{C_0}{V_2} sin\Theta_2$$

- The speed of light divided by the velocity of light in the medium is the ratio of light travel (n). With that we get, the Snell's Law equation.

$$n_1 sin\Theta_1 = n_2 sin\Theta_2$$

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Practicality of Snell's Law

- This equation can be used like the following: With one factor double the other and the angle is 45º,

$$n_1 sin\Theta_1 = n_2 sin\Theta_2$$

$$(1)\sin(45) = (2) sin\Theta_2$$

$$\sin^{-1}\sin\Theta_2 = \sin^{-1}\frac{\sin(45)}{2}$$

$$\Theta = 20.705^0$$

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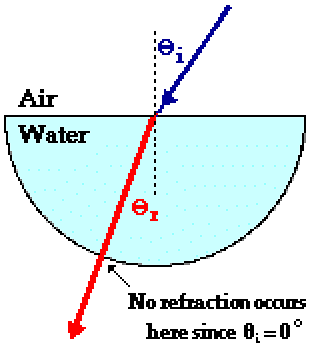
Errors in Snell's Law

- Light bends by slowing down. The way that light slows down is by changing mediums. The reason that the light continues to go "straight" is that the light will still go in the path that takes the least time. The only situations that Snell's Law "breaks down" would be when w=the angle of incidence is 0º or when the speed of light ratios are the same. $(n_1 = n_2)$



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Conclusion

- Snell's Law along with Fermat's Principle are used to determine certain characteristics of light. When light is going from medium to medium, the path with always go the path of least time traveled. However, because of the density and other factors of the medium change, the light's velocity will slow down.

Works Cited

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