Snell Law: Refraction

Calculus I Project

The purpose of this project is to try to understand the **refraction of light**. Refraction is the bending of light that takes place when it travels from one medium (such as air) into another (such as water). Refraction is the reason why a straight stick appears bent when it is held halfway submerged in water. It's also the reason why sunlight bounces off windows at some angles and not at others.

A great deal about the way light behaves when it reflects off mirrors, or goes through lenses can be deduced from a seemingly simple principle which was discovered by Pièrre de Fermat in 1658 (this is the same Fermat who is famous for the problem known as "Fermat's Last Theorem", the proof of which was finally published in 1995). Fermat's Principle for light states that:

A beam of light will always follow the path which makes its traveling time shortest.

Carry out the following activities.

A. Light always moves at a steady speed while it stays in the same medium. For example, in air, light zips along at 2.998×10^8 meters per second, but when traveling through water, its speed is a mere $2.254 \times 10^8 \mathrm{ms}^{-1}$.

Can you explain why one consequence of Fermat's Principle is that light traveling through just one medium must always travel in a straight line?

B. Suppose that the x-axis runs along the surface of a pool of water and that a beam of light is shone from the point A at $(1, 1)$ to the point B at $(-1, -1)$. Draw a picture of the path the beam takes. If the beam of light is to obey Fermat's Principle, find the coordinates of the point C where it must cross from the air into the water. (Your solution should be justified with appropriate mathematics.)

C. Next suppose that the x-axis is the boundary between two media which light can move through, but not necessarily water and air. Suppose the speed of light in the top one is c_1 and in the bottom one is c_2. Again, you are to suppose that a beam of light travels from A to B. You should find an equation which you could solve to find the coordinates of C.

D. The problem with the equation which you get in the last part is that it is very hard to solve algebraically. So let's concentrate on the angle θ_1 which AC makes with the x-axis and the angle θ_2 that CB makes with the x-axis.

Using geometry and your equation from Part C., find a relationship between θ_1 and θ_2.

Demonstrate your relationship with specific values of θ_1, θ_2 or c_1, c_2 (that you choose) for which you can predict the answers you should get by common sense.

E. Snell's Law says that for light traveling from any point A in one medium to any point B in another medium

$$\frac{\cos\theta_1}{c_1} = \frac{\cos\theta_2}{c_2}$$

where c_1 is the speed of light in the first medium and c_2 is the speed of light in the second medium. The angles θ_1 and θ_2 are as shown in the picture[1].

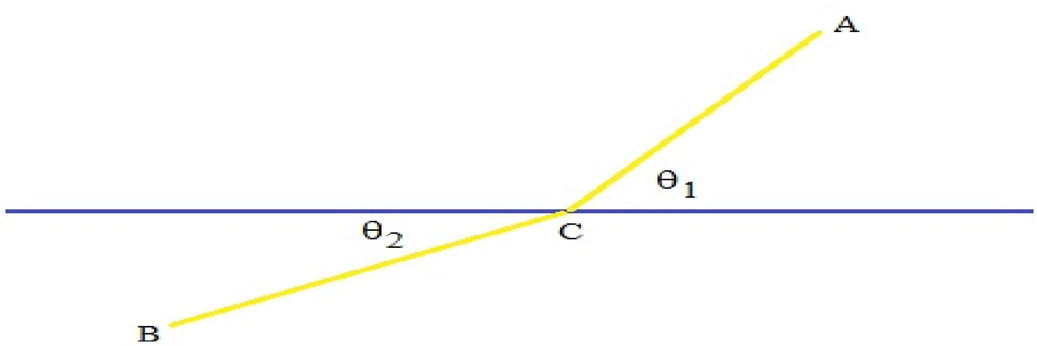

Figure 1: Light traveling from point A to point B with change of medium at the interface occurring at point C.

Using the ideas you used in Part D., prove Snell's Law.

Suppose the speed of light in the left hand medium is twice as large as the speed of light in the right hand medium. If a ray of light hits the interface at $45°$, what is the angle it is refracted to? Include a diagram with your discussion of the answer to this question.

F. Finally go back to the case where we're thinking about light traveling from air into water. Describe in words how the light bends. (Not the exact angles, just a qualitative explanation of how it bends.) Can you see a situation where Snell's Law breaks down? What do you think happens to the light then?

[1] You may find Snell's Law stated differently, but this is correct based on the angles as they are defined here.