

Solving Trigonometric Equations (Challenge Problems)

Solve each trigonometric equation on the indicated interval. (Solutions are on the next page.)

(a) $\tan(4x) = 1$ on $[0, 2\pi)$

(b) $\sin^2 x - \frac{5}{2} \sin x + 1 = 0$ on $[0, 2\pi)$

(c) $\sec^2 x - 4 = 0$ on $[0, 2\pi)$

(d) $\sec^2 x - 4 = 0$, all solutions on $(-\infty, \infty)$

(e) $\cos(2x) = -\sin(2x)$ on $[0, 2\pi)$

(f) $(\cos x + \frac{1}{3})(\sin x - \frac{1}{7}) = 0$, on $[0, 2\pi)$

Here are the solutions.

$$(a) \left\{ \frac{\pi}{16}, \frac{5\pi}{16}, \frac{9\pi}{16}, \frac{13\pi}{16}, \frac{17\pi}{16}, \frac{21\pi}{16}, \frac{25\pi}{16}, \frac{29\pi}{16} \right\}$$

$$(b) \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$

$$(c) \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$$

$$(d) \left\{ \frac{\pi}{3} + n\pi, \frac{2\pi}{3} + n\pi \mid n \text{ any integer} \right\}$$

$$(e) \left\{ \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8} \right\}$$

$$(f) \left\{ \cos^{-1} \left(-\frac{1}{3} \right), 2\pi - \cos^{-1} \left(-\frac{1}{3} \right), \sin^{-1} \left(\frac{1}{7} \right), \pi - \sin^{-1} \left(\frac{1}{7} \right) \right\}$$