IDs and Other Trig Related Things You Need to Know

(1) Reciprocal IDs:

$$\cot\theta = \frac{1}{\tan\theta}, \quad \csc\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}$$

(2) Quotient IDs:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

(3) Pythagorean IDs:

$$\sin^2\theta + \cos^2\theta = 1, \quad \tan^2\theta + 1 = \sec^2\theta, \quad 1 + \cot^2\theta = \csc^2\theta$$

(4) Basic Definitions (right triangle trig)

$$\sin\theta = \frac{opp}{hyp}, \quad \cos\theta = \frac{adj}{hyp}, \quad \tan\theta = \frac{opp}{adj}$$

(5) Special Angle Trig Values:

θ^0	0^0	30^0	45^0	60^0	90^0
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef

(6) Quadrantal Angle Trig Values:

θ^0	0^0	90^0	180^0	270^0
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\theta$	0	1	0	-1
$\cos\theta$	1	0	-1	0
$\tan\theta$	0	undef	0	undef
$\csc\theta$	undef	1	undef	-1
$\sec\theta$	1	undef	-1	undef
$\cot\theta$	undef	0	undef	0

(7) Signs of Trig Functions by Quadrant:

Quad I **(A)** All are positive.

Quad II **(S)** Sine and Cosecant are positive, all else negative.

Quad III **(T)** Tangent and Cotangent are positive, all else negative.

Quad IV **(C)** Cosine and Secant are positive, all else are negative.

(Pneumonic: **A**ll **S**tudents **T**ake **C**alculus. Taken counter-clockwise, i.e. the positive direction.)

(8) If θ is an angle in standard position whose terminal side is in any of the four quadrants, and α is the reference angle for θ, then

$$\sin \theta = \pm \sin \alpha, \quad \cos \theta = \pm \cos \alpha, \quad \tan \theta = \pm \tan \alpha,$$

$$\csc \theta = \pm \csc \alpha, \quad \sec \theta = \pm \sec \alpha, \quad \cot \theta = \pm \cot \alpha.$$

The sign is determined by the quadrant that θ's terminal side is in.

(9) Cofunction Identities:

$$\sin \left(\frac{\pi}{2} - \theta \right) = \cos \theta, \quad \cos \left(\frac{\pi}{2} - \theta \right) = \sin \theta,$$

$$\csc \left(\frac{\pi}{2} - \theta \right) = \sec \theta, \quad \sec \left(\frac{\pi}{2} - \theta \right) = \csc \theta,$$

$$\tan \left(\frac{\pi}{2} - \theta \right) = \cot \theta, \quad \cot \left(\frac{\pi}{2} - \theta \right) = \tan \theta$$

(10) Periodicity: For every integer n

$$\sin(x + 2\pi n) = \sin(x), \quad \cos(x + 2\pi n) = \cos(x),$$

$$\csc(x + 2\pi n) = \csc(x), \quad \sec(x + 2\pi n) = \sec(x),$$

$$\tan(x + \pi n) = \tan(x), \quad \cot(x + \pi n) = \cot(x)$$

(11) Even and Odd Symmetry:

$$\sin(-x) = -\sin(x), \quad \cos(-x) = \cos(x),$$

$$\csc(-x) = -\csc(x), \quad \sec(-x) = \sec(x),$$

$$\tan(-x) = -\tan(x), \quad \cot(-x) = -\cot(x)$$

(12) Sum and Difference:

$$\sin(u \pm v) = \sin u \cos v \pm \sin v \cos u,$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin v \sin u,$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v},$$

(13) Double Angle:

$$
\begin{aligned}
\sin(2u) &= 2 \sin u \cos u \\
\cos(2u) &= \cos^2 u - \sin^2 u \\
&= 2 \cos^2 u - 1 \\
&= 1 - 2 \sin^2 u \\
\tan(2u) &= \frac{2 \tan u}{1 - \tan^2 u}
\end{aligned}
$$

(14) Half-Angle & Power Reducing:

$$\sin^2 x = \frac{1 - \cos 2x}{2}, \quad \sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}, \quad \cos \frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}, \quad \tan \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

The sign for the half angle identity is determined by the quadrant of the terminal side of $\frac{x}{2}$ in standard position.