Exam II Math 2253H sec. 5H

Fall 2014

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ________________________________

Problem	Points
1	
2	
3	
4	
5	
6	

INSTRUCTIONS: There are 6 problems. The point value for each problem is listed with the problem. There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam and may result in removal from this class.** To receive full credit, you must clearly justify your answer and use proper notation.

(1) Consider the relation $x^4 - 2xy^2 = 1 + \sin y$.

(a) (10 points) Find $\frac{dy}{dx}$ using implicit differentiation.

$$4x^3 - 2y^2 - 4xy\frac{dy}{dx} = \cos y \frac{dy}{dx}$$

$$4x^3 - 2y^2 = (\cos y + 4xy)\frac{dy}{dx}$$

$$\Longrightarrow \quad \frac{dy}{dx} = \frac{4x^3 - 2y^2}{\cos y + 4xy}$$

(b) (5 points) Find the equation of the line tangent to the graph of the curve at the point $(1, 0)$. Express your answer in the form $y = mx + b$.

The slope $m = \left.\frac{dy}{dx}\right|_{(1,0)} = \frac{4-0}{1+0} = 4$

$$y - 0 = 4(x-1)$$

$$\Longrightarrow \quad y = 4x - 4$$

(2) (15 points) Find all the points on the graph of f at which the tangent line is horizontal. The x-value for each point is sufficient. (Hint: There are three such points.)

$f(x) = (x-2)^3(x+1)^2$

The tangent is horizontal if the derivative is zero.

$$f'(x) = 3(x-2)^2(x+1)^2 + 2(x-2)^3(x+1)$$

$$= (x-2)^2(x+1)\left[3(x+1) + 2(x-2)\right]$$

$$= (x-2)^2(x+1)(5x-1)$$

$$f'(x) = 0 \implies (x-2)^2(x+1)(5x-1) = 0$$

$$\implies x = 2, \quad x = -1, \quad \text{or} \quad x = \frac{1}{5}$$

(3) (30 points, 10 each) Evaluate the derivative of each function. Do not leave compound fractions in your answer; otherwise it is not necessary to simplify.

(a) $f(x) = 3x^3 \tan(4x)$

$$f'(x) = 9x^2 \tan(4x) + 3x^3 \sec^2(4x) \cdot 4$$

$$= 9x^2 \tan(4x) + 12x^3 \sec^2(4x)$$

(b) $g(t) = \sec\left(\sqrt{t}\right)$

$$g'(t) = \sec(\sqrt{t}) \tan(\sqrt{t}) \cdot \frac{1}{2\sqrt{t}}$$

$$= \frac{\sec(\sqrt{t}) \tan(\sqrt{t})}{2\sqrt{t}}$$

(c) $r(\theta) = \sin\left(\frac{1}{\theta^2}\right)$ $= \sin(\theta^{-2})$

$$r'(\theta) = \cos(\theta^{-2})(-2\theta^{-3})$$

$$= \frac{-2\cos\left(\frac{1}{\theta^2}\right)}{\theta^3}$$

(4) (10 points) If $p(x)$ is the total value of the production when there are x workers in a plant, then the *average productivity* of the workforce of the plant is

$$A(x) = \frac{p(x)}{x}.$$

Find $A'(x)$. ($p'(x)$ will be part of of your answer.) Why does the company want to hire more workers is $A'(x) > 0$?

$$A'(x) = \frac{p'(x)x - p(x)}{x^2}$$

If $A'(x) > 0$, then average productivity is increasing with increasing x.

Hence increasing the number of workers (i.e. hiring) will result in greater average productivity.

(5) (15 points) Find the equation of the line tangent to the graph of $f(x) = 2x^3 - x^2 - 4$ at the point $(2, f(2))$.

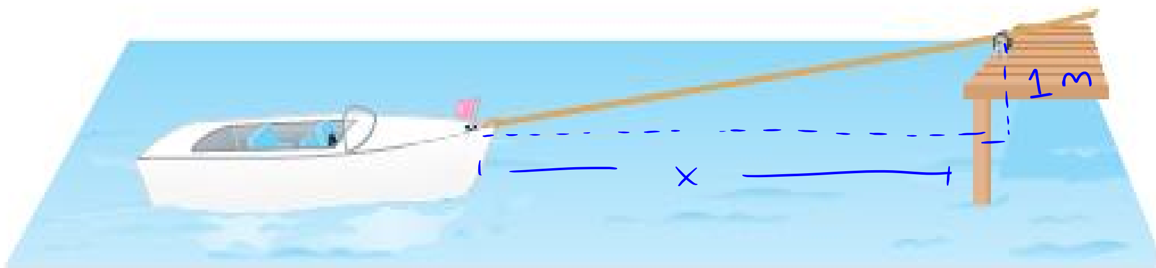
$f'(x) = 6x^2 - 2x$, $f(2) = 2 \cdot 2^3 - 2^2 - 4 = 16 - 8 = 8$

$f'(2) = 6 \cdot 2^2 - 2 \cdot 2 = 24 - 4 = 20$

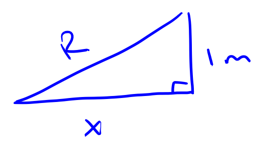
$y - 8 = 20(x - 2)$

$y = 20x - 12$

(6) (15 points) A boat is pulled into a dock by a rope attached to the bow of the boat passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/sec, how fast is the boat approaching the dock when it is 8 m from the dock?



Let x be the boats distance from the dock and R the length of the rope.



Given: $\frac{dR}{dt} = -1 \frac{m}{sec}$

Note R is decreasing.

Also, $R^2 = x^2 + 1^2$.

Q: $\frac{dx}{dt} = ?$ when $x = 8$ m

$$2R\frac{dR}{dt} = 2x\frac{dx}{dt} \implies \boxed{\frac{dx}{dt} = \frac{R}{x}\frac{dR}{dt}}$$

When $x = 8$ m, $R^2 = (8m)^2 + (1m)^2 = 64m^2 + 1m^2 = 65 m^2$

So when $x = 8$ m

$$\frac{dx}{dt} = \frac{\sqrt{65}\ m}{8\ m} \cdot (-1)\frac{m}{sec} = \frac{-\sqrt{65}}{8}\frac{m}{sec}.$$

The boat is approaching the dock at a rate of $\frac{\sqrt{65}}{8}$ m/sec.