Exam III Math 2253H sec. 5H

Fall 2014

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	

INSTRUCTIONS: There are seven problems. The point value for each problem is listed with the problem. There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam and may result in removal from this class.** To receive full credit, you must clearly justify your answer and use proper notation.

(1) (15 points) A polynomial f has the given second derivative.

$$f''(x) = 2(x-1)^2(x+4)(x-5).$$

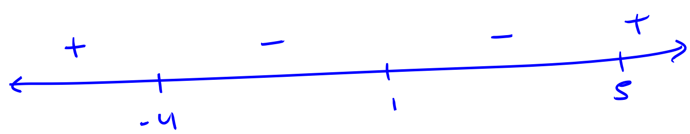
Determine the intervals on which f is concave up and the intervals on which f is concave down.

f'' is always well defined.

$f''(x) = 0 \implies 0 = 2(x-1)^2(x+4)(x-5)$

$\implies x = 1$, $x = -4$ or $x = 5$

Sign of f''



$f''(-5)$ $(+)(-)(-)$ $\qquad$ $f''(2)$ $(+)(+)(-)$

$f''(0)$ $(+)(+)(-)$ $\qquad$ $f''(6)$ $(+)(+)(+)$

f is concave up on $(-\infty, -4) \cup (5, \infty)$.

f is concave down on $(-4, 1) \cup (1, 5)$.

(2) (15 points) Suppose f is a rational function with domain $\{x \mid x \neq 4\}$ and whose first derivative is

$$f'(x) = \frac{x(x+2)^2}{x-4}.$$

(a) Find all critical numbers of f. (Remember to consider the domain.)

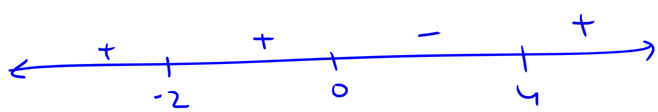
$f'(x) = 0 \Rightarrow x(x+2)^2 = 0 \Rightarrow x = 0$ or $x = -2$

$f'(x)$ UND $\Rightarrow x - 4 = 0 \Rightarrow x = 4$ (not in the domain)

f has two critical numbers 0 and -2.

(b) Determine the intervals on which f is increasing and the intervals on which f is decreasing.

Sign of f'



$f'(-3)$ $\frac{(-)(+)}{-}$ $\qquad$ $f'(1)$ $\frac{(+)(+)}{-}$

$f'(-1)$ $\frac{(-)(+)}{-}$ $\qquad$ $f'(5)$ $\frac{(+)(+)}{+}$

f is increasing on $(-\infty, -2) \cup (-2, 0) \cup (4, \infty)$.

f is decreasing on $(0, 4)$.

(3) (10 points) For the given function and the specified interval, find the value of c that satisfies the conclusion of the Mean Value Theorem.

$$f(x) = \cos x + \sin x \quad \text{on} \quad \left[0, \frac{\pi}{2}\right]$$

$f'(x) = -\sin x + \cos x$

$$\left.\begin{array}{l} f\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right) = 0+1 = 1 \\ f(0) = \cos(0) + \sin(0) = 1+0 = 1 \end{array}\right\} \Rightarrow \frac{f\left(\frac{\pi}{2}\right) - f(0)}{\frac{\pi}{2} - 0} = 0$$

$$f'(c) = \frac{f\left(\frac{\pi}{2}\right) - f(0)}{\frac{\pi}{2} - 0} \Rightarrow -\sin c + \cos c = 0$$

$$\Rightarrow \sin c = \cos c$$

There is one such value on $\left[0, \frac{\pi}{2}\right]$ namely

$c = \frac{\pi}{4}$.

(4) (15 points) Let $f(x) = \sqrt{x} - \frac{x}{4}$ for $x > 0$. Find the critical number of f and classify it as a local maximum or a local minimum.

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{4}$ $\qquad$ $f'(x)$ is defined for $x > 0$

$$f'(x) = 0 \Rightarrow \frac{1}{2\sqrt{x}} - \frac{1}{4} = 0 \Rightarrow \frac{1}{2\sqrt{x}} = \frac{1}{4}$$

$$\Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$$

$$f''(x) = \frac{1}{2}\left(\frac{-1}{2}\right) x^{-3/2} = \frac{-1}{4\sqrt{x^3}}$$

$$f''(4) = \frac{-1}{4\sqrt{4^3}} = \frac{-1}{32} < 0$$ concave down

f takes a local maximum @ $x = 4$.

(5) (15 points) A particle moves along the x-axis so that its acceleration at time t is given by

$$a(t) = 12t^2 - \cos(t), \quad \text{m/sec}^2.$$

At time $t = 0$, the velocity v and position s of the particle are known to be

$$v(0) = 1 \text{ m/sec} \quad \text{and} \quad s(0) = 0 \text{ m}$$

Determine the position $s(t)$ of the particle for all $t > 0$.

$a(t) = v'(t) \Rightarrow v(t) = 12\frac{t^3}{3} - \sin t + C$

$v(t) = 4t^3 - \sin t + C$

$v(0) = 1 = 4 \cdot 0 - \sin 0 + C \Rightarrow C = 1$

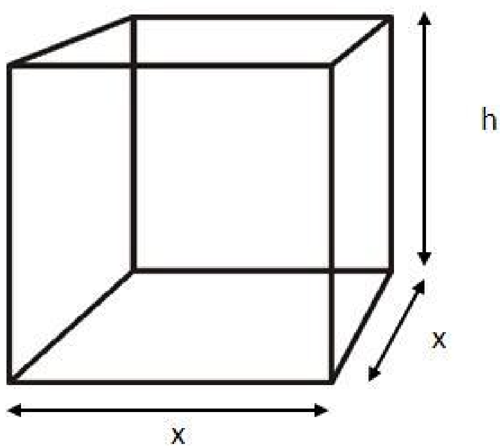
$v(t) = s'(t) \Rightarrow s(t) = 4 \cdot \frac{t^4}{4} + \cos t + t + k$

$s(t) = t^4 + \cos t + t + k$

$s(0) = 0 = 0 + \cos 0 + 0 + k \Rightarrow k = -\cos(0) = -1$

$\boxed{s(t) = t^4 + \cos t + t - 1}$

(6) (15 points) A closed rectangular box with a square base is to be constructed out of metal. The material for the top and bottom (the square sides) costs \$2.00 per square inch, and the material for the remaining four sides costs \$1.00 per square inch. The box must have a volume of 16 cubic inches. Find the dimensions of the box that will minimize the cost of material.



Take x, h in inches

Volume

$V = x^2 h$

$V = 16 \text{ in}^3 \Rightarrow$

$h = \frac{16}{x^2}$

Top + bottom

Area $= 2x^2 \text{ in}^2$

Cost $= 2x^2 \text{ in}^2 \cdot \$2 / \text{in}^2$

$= 4x^2$ \$

Lateral Surface

Area $= 4xh \text{ in}^2$

Cost $= 4xh \text{ in}^2 \cdot \$1 / \text{in}^2$

$= 4xh$ \$

Total cost

$C = 4x^2 + 4xh$ \$

$C(x) = 4x^2 + 4x\left(\frac{16}{x^2}\right) = 4x^2 + \frac{64}{x}$

Crit # : $C'(x) = 8x - \frac{64}{x^2}$ Note $x > 0$

$C'(x) = 0 \Rightarrow 8x = \frac{64}{x^2} \Rightarrow x^3 = \frac{64}{8} = 8$

$\Rightarrow x = 2$

$C''(x) = 8 + \frac{128}{x^3}$, $C''(2) = 8 + \frac{128}{2^3} > 0$ so C is minimum when $x = 2$

$h = \frac{16}{x^2} = \frac{16}{4} = 4$.

The dimensions to minimize cost are

$2 \text{ in} \times 2 \text{ in} \times 4 \text{ in}$

(7) (15 points) The graph of the function f has two horizontal asymptotes. Find the equations of the horizontal asymptotes.

$$f(x) = \frac{\sqrt{x^2 + 2x}}{4x - 1}$$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{\sqrt{x^2 + 2x}}{4x - 1} \cdot \frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{x}}$$

for $x > 0$ $\sqrt{x^2} = x$

$$= \lim_{x \to \infty} \frac{\sqrt{1 + 2/x}}{4 - \frac{1}{x}} = \frac{\sqrt{1}}{4} = \frac{1}{4}$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{\sqrt{x^2 + 2x}}{4x - 1} \cdot \frac{\frac{-1}{\sqrt{x^2}}}{\frac{1}{x}}$$

for $x < 0$ $x = -\sqrt{x^2}$

$$= \lim_{x \to -\infty} \frac{-\sqrt{1 + 2/x}}{4 - \frac{1}{x}} = \frac{-\sqrt{1}}{4} = \frac{-1}{4}$$

The equations of the two horizontal asymptotes are

$y = \frac{1}{4}$ and $y = \frac{-1}{4}$.