Exam IV Math 2253H sec. 5H

Fall 2014

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ____________________

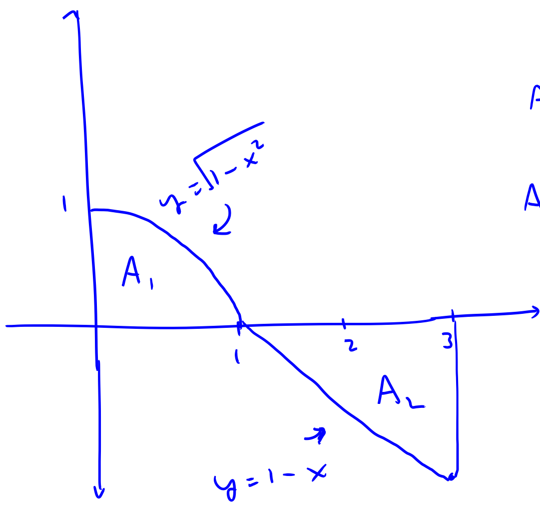
Problem	Points
1	
2	
3	
4	
5	
6	
7	

INSTRUCTIONS: There are seven problems. The point value of each problem is listed with the problem. There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam and may result in removal from this class.** To receive full credit, you must clearly justify your answer and use proper notation.

(1) (10 points) Evaluate the definite integral by interpreting it in terms of areas.

$$\int_0^3 f(x)\,dx \quad \text{where} \quad f(x) = \begin{cases} \sqrt{1-x^2}, & 0 < x \le 1 \\ 1-x, & x > 1 \end{cases}$$

$$y = \sqrt{1-x^2} \implies y^2 = 1-x^2 \implies x^2+y^2 = 1$$



$$A_1 = \frac{1}{4}\pi(1^2) = \frac{\pi}{4}$$

$$A_2 = \frac{1}{2}(2)(2) = 2$$

$$\int_0^3 f(x)\,dx = A_1 - A_2$$

$$= \frac{\pi}{4} - 2$$

(2) (20 points) Evaluate the indefinite integral using any method.

$$\int (3x+2)^2\,dx = \int (9x^2 + 12x + 4)\,dx$$

$$= 3x^3 + 6x^2 + 4x + C$$

(3) (20 points) Evaluate the definite integral using any method.

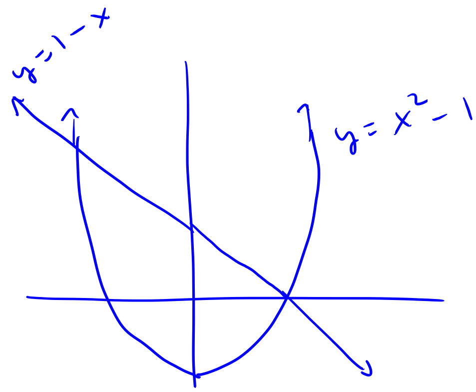
$$\int_1^8 \frac{2}{\sqrt[3]{x}}\,dx = 2\int_1^8 x^{-1/3}\,dx$$

$$= 2\left.\frac{x^{2/3}}{2/3}\right|_1^8 = \left.3x^{2/3}\right|_1^8$$

$$= 3(8)^{2/3} - 3(1)^{2/3} = 3(4) - 3 = 9$$

(4) (15 points) Find the area of the region bounded between the curves

$$y = x^2 - 1 \quad \text{and} \quad y = 1 - x$$



They intersect when

$$x^2 - 1 = 1 - x$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2 \quad \text{or} \quad x = 1$$

The line is the top curve on $-2 \leq x \leq 1$

$$A = \int_{-2}^{1} (1 - x - (x^2 - 1))\, dx$$

$$= \int_{-2}^{1} (2 - x - x^2)\, dx$$

$$= 2x - \frac{x^2}{2} - \frac{x^3}{3} \Big|_{-2}^{1}$$

$$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right)$$

$$= 2 - \frac{1}{2} - \frac{1}{3} + 6 - \frac{8}{3}$$

$$= 8 - \frac{1}{2} - \frac{9}{3} \qquad = 5 - \frac{1}{2}$$

$$= \frac{9}{2}$$

(5) (10 points) Evaluate the limit by writing it as an integral[1], and evaluating the resulting integral in the usual way using the Fundamental Theorem. (Hint: Take $a = 0$.)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2 \frac{2}{n}$$

If $a=0$, $\frac{b-a}{n} = \frac{2}{n} \Rightarrow b=2$

$f(x_i) = \left(\frac{2i}{n}\right)^2$

$x_i = a + i\Delta x$

$\Delta x = \frac{2}{n}$

$= i\left(\frac{2}{n}\right) = \frac{2i}{n}$

Since $f(x_i) = \left(\frac{2i}{n}\right)^2 = x_i^2$

we can take $f(x) = x^2$

$$\int_0^2 x^2\,dx = \frac{x^3}{3}\Big|_0^2 = \frac{8}{3} - 0 = \frac{8}{3}$$

Hence

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2 \frac{2}{n} = \frac{8}{3}$$

[1]Recall that if f is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x, \quad \text{where} \quad \Delta x = \frac{b-a}{n} \quad \text{and} \quad x_i = a + i\Delta x.$$

(6) (15 points) Evaluate the indefinite integral using any method.

$$\int x\sqrt[3]{x-1}\,dx$$

let $u = x-1$, $du = dx$ and $x = u+1$

$$= \int (u+1)\, u^{1/3}\, du = \int \left(u^{4/3} + u^{1/3}\right) du$$

$$= \frac{u^{7/3}}{7/3} + \frac{u^{4/3}}{4/3} + C$$

$$= \frac{3}{7} u^{7/3} + \frac{3}{4} u^{4/3} + C$$

$$= \frac{3}{7} \sqrt[3]{(x-1)^7} + \frac{3}{4} \sqrt[3]{(x-1)^4} + C$$

(7) (10 points) Find $g'(x)$ where

$$g(x) = \int_{2x}^{x^2+1} \sec(\sqrt{t})\, dt$$

$$g'(x) = \sec\left(\sqrt{x^2+1}\right)(2x) - \sec\left(\sqrt{2x}\right)\cdot 2$$

$$= 2x \sec\left(\sqrt{x^2+1}\right) - 2\sec\left(\sqrt{2x}\right)$$