Final Exam Math 2254 sec. 10

Fall 2013

Name: (2 points) w/ Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ____

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	

INSTRUCTIONS: There are 8 problems worth 14 points each. Only 7 problems will be counted for credit. You may eliminate any one problem, or I will count your best 7 out of 8. **No** use of notes, books, or calculators is allowed. Cell phones, tablets, or other app. running, internet access devices must be put away. **Illicit use of notes, a calculator, a phone or app. running device will result in a grade of zero for this exam and may result in removal from this class.** To receive full credit, you must follow the directions given and clearly justify your answer. Answers without supporting work will not be considered for credit.

(1) Evaluate the derivative of each function. (Do not leave compound fractions in your final answer.)

(a) $y = 3^x \ln(4x)$

$$\frac{dy}{dx} = (\ln 3)\, 3^x \ln(4x) + 3^x \frac{4}{4x}$$

$$= (\ln 3)\, 3^x \ln(4x) + \frac{3^x}{x}$$

(b) $g(x) = \sin^{-1}(x-2)$

$$g'(x) = \frac{1}{\sqrt{1-(x-2)^2}}$$

(2) Evaluate each indefinite integral.

(a) $\displaystyle\int \frac{4x+1}{x^2-x-2}\,dx$

Partial Fractions

$$\frac{4x+1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

$$4x+1 = A(x+1) + B(x-2)$$

$x=2 \qquad 9 = 3A \quad \Rightarrow A = 3$

$x=-1 \qquad -3 = -3B \quad \Rightarrow B = 1$

$$\int \frac{4x+1}{x^2-x-2}\,dx = \int \left(\frac{3}{x-2} + \frac{1}{x+1}\right) dx =$$

$$3\ln|x-2| + \ln|x+1| + C$$

(b) $\displaystyle\int \cos^3\theta\,d\theta = \int \cos^2\theta \cos\theta\,d\theta$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= \int (1-\sin^2\theta)\cos\theta\,d\theta$$

$$= \int (1-u^2)\,du$$

$$= u - \frac{u^3}{3} + C$$

$$= \sin\theta - \frac{1}{3}\sin^3\theta + C$$

(3) Evaluate each limit. Use proper notation and indicate any special rules/theorems used.

(a) $\lim_{x \to 0} \dfrac{x^2 - 4x}{\sin(2x)}$ $= \text{"} \dfrac{0}{0} \text{"}$ apply l'H rule

$= \lim_{x \to 0} \dfrac{2x-4}{2\cos(2x)} = \dfrac{-4}{2} = -2$

(b) $\lim_{x \to 0^+} e^{\frac{1}{x}}$

$\lim_{x \to 0^+} \dfrac{1}{x} = +\infty$

$= \infty$

and $e^t \to \infty$ as $t \to \infty$

(4) Find the Taylor polynomial of degree 2 centered at the given value of a for the indicated function f.

$$f(x) = x^{1/4}, \quad a = 1$$

$f(x) = x^{1/4}$

$f'(x) = \frac{1}{4} x^{-3/4}$

$f''(x) = \frac{-3}{16} x^{-7/4}$

$f(1) = 1$

$f'(1) = \frac{1}{4}$

$f''(1) = \frac{-3}{16}$

$$T_2(x) = \frac{f(1)}{0!} + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2$$

$$T_2(x) = 1 + \frac{1}{4}(x-1) - \frac{3}{32}(x-1)^2$$

(5) Find the equation of the line tangent to the graph of the parametric curve at the indicated value of the parameter.

$$x = 3t^2 + 1, \quad y = t^3 - 6t, \quad t = 1$$

When $t=1$, $x = 3+1 = 4$ $\quad y = 1-6 = -5$

$\frac{dx}{dt} = 6t$, $\frac{dy}{dt} = 3t^2 - 6$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3t^2-6}{6t}$$

When $t=1$, the tangent line has slope

$$m = \left.\frac{dy}{dx}\right|_{t=1} = \frac{3-6}{6} = \frac{-1}{2}$$

The line is

$$y - (-5) = \frac{-1}{2}(x-4)$$

$$y = \frac{-1}{2}x - 3$$

(6) (a) Determine if the integral is proper or improper. If improper, state why.

$$\int_0^1 \ln x \, dx$$

It's improper. The natural log has vertical asymptote $x=0$.

(b) Evaluate the integral if possible.

$$\int_0^1 \ln x \, dx = \lim_{t \to 0^+} \int_t^1 \ln x \, dx \quad *$$

$$= \lim_{t \to 0^+} x \ln x - x \Big|_t^1$$

$$= \lim_{t \to 0^+} \left[1 \ln 1 - 1 - (t \ln t - t) \right]$$

$$= \lim_{t \to 0^+} \left(-1 - t \ln t + t \right) \quad **$$

$$= -1 - 0 + 0 = -1$$

$$* \int \ln x \, dx = x \ln x - \int dx = x \ln x - x + C$$

$u = \ln x$ $\quad du = \frac{1}{x} dx$

$dv = dx$ $\quad v = x$

$$** \lim_{t \to 0^+} t \ln t = \lim_{t \to 0^+} \frac{\ln t}{1/t} = \text{“}\frac{\infty}{\infty}\text{”}$$ use l'H rule

$$= \lim_{t \to 0^+} \frac{\frac{1}{t}}{-1/t^2} = \lim_{t \to 0^+} -t = 0$$

(7) Determine any points on the curve defined by the parametric equations where the tangent line would be (a) horizontal, and (b) vertical. Give your answers as ordered pairs (x, y).

$$x = te^t, \quad y = 3t^4 - 4t^3 + 1$$

$\frac{dx}{dt} = e^t + te^t$ $\quad\quad$ $\frac{dy}{dt} = 12t^3 - 12t^2$

$= e^t(1+t)$ $\quad\quad$ $= 12t^2(t-1)$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{12t^2(t-1)}{e^t(t+1)}$$

A tangent is horizontal if $\frac{dy}{dx} = 0$

$12t^2(t-1) = 0 \Rightarrow t = 0$ or $t = 1$

A tangent line is vertical if $\frac{dy}{dx}$ is undefined (looks like $\frac{\text{constant}}{0}$)

$e^t(t+1) = 0 \Rightarrow e^t = 0$ no soln. or $t = -1$

@ $t = 0$, $x = 0$ $\quad y = 1$

$t = 1$, $x = e$ $\quad y = 0$

$t = -1$, $x = -e^{-1}$ $\quad y = 8$

The tangent line is horizontal @ $(0, 1)$ and $(e, 0)$.

The tangent line is vertical @ $(\frac{-1}{e}, 8)$.

(8) Find and classify all local extrema of the function $f(x) = x^2e^x$. (That is, find any local maximum and minimum values of f and the x-value(s) where such occur. Justify your claim with any appropriate tests.)

$$f'(x) = 2xe^x + x^2e^x = xe^x(2+x)$$

critical numbers: $f'(x) = 0 \Rightarrow x = 0$ or $x = -2$

f' is never undefined.

$$f''(x) = 2e^x + 2xe^x + 2xe^x + x^2e^x$$
$$= e^x(2+4x+x^2)$$

2nd derivative test:

$f''(0) = 2e^0 = 2 > 0$ concave up

$f''(-2) = e^{-2}(2-8+4) = -2e^{-2} < 0$ concave down.

f has a local minimum @ $(0, f(0)) = (0,0)$

f has a local maximum @ $(-2, f(-2)) = (-2, 4e^{-2})$