INSTRUCTIONS: There are 7 problems worth 14 points each. You may use your text book (Atkinson & Han), one 8.5” by 11” page of written notes and a calculator. NO use of cell phones, tablets or any app running, internet access device may be used. Illicit use of such items will result in a grade of zero on this exam. To receive full credit, you must follow the directions given and clearly justify your answer.
(1) Use the method of undetermined coefficients to find an approximation formula $D_h f(x)$

to the first derivative of the form

$$f'(x) \approx D_h f(x) = Af(x + 2h) + Bf(x - h)$$
(2) Find an $LU$ decomposition for the matrix $A$. (Supporting work must be provided to receive credit.)

$$A = \begin{bmatrix}
-1 & 2 & 2 \\
-1 & 6 & 1 \\
-3 & -2 & 11
\end{bmatrix}$$
(3) Use Gaussian elimination with back substitution to solve the system of equations. (Supporting work must be provided to receive credit.)

\[
\begin{align*}
    x_1 - 2x_2 + x_3 &= -5 \\
    2x_1 + x_2 &= 0 \\
    x_1 - x_2 + 2x_3 &= -3
\end{align*}
\]
(4) Use the Gaussian numerical integration formula $I_2(f)$ to approximate the integral. Give your answer with 5 digits to the right of the decimal place. Do not give your answer in scientific notation.

$$\int_{-1}^{1} \frac{\sin t}{t} dt$$
(5) Find the Taylor polynomial $P_2(x)$ of degree 2 centered at $a = 8$ with the remainder $R_2(x)$ for the function

$$f(x) = \sqrt{x}.$$ 

Use the remainder to find a bound on the absolute value of the error $|\sqrt{8.1} - P_2(8.1)|$. Give 8 digits to the right of the decimal; do not give your answer in scientific notation.
(6) We wish to compute $\sqrt[3]{4}$ by finding the real root of the function $f(x) = x^3 - 4$. Set up the Newton iteration formula. Simplify and give the formula in the space provided.

Starting with $x_0 = 1$, fill in the table. Show 5 digits to the right of the decimal; do not leave answers in scientific notation.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$x_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.00000</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Iteration Formula:

$x_{n+1} = \underline{\text{____________}}$
(7) The number of students attending SPSU in 2006 was 4500. In 2013, the number of students at SPSU is 6200. Use a linear interpolation to estimate the number of students attending SPSU in 2010. Round your answer to the nearest integer.