

Exam 1 Math 1112 sec. 54 Spring 2019

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: _____

Problem	Points
1	
2	
3	
4	
5	
6	
Total	

INSTRUCTIONS: There are 6 problems (some with multiple parts). The point values are listed with the problems; there are 85 total points. There are no notes or books, allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, and written using proper notation.

1. Consider the function $f(x) = 2|x| - x^2$. Evaluate each expression. Your final response should be simplified to the extent possible. (2 points each)

(a) Evaluate $f(3) = 2|3| - 3^2 = 6 - 9 = -3$

(b) Evaluate $f(-2) = 2|-2| - (-2)^2 = 4 - 4 = 0$

(c) Evaluate $f(2a) = 2|2a| - (2a)^2 = 4|a| - 4a^2$

(d) Evaluate $f(x-2) = 2|x-2| - (x-2)^2 = 2|x-2| - (x^2 - 4x + 4)$
 $= 2|x-2| - x^2 + 4x - 4$

(e) Evaluate $f(x+h) = 2|x+h| - (x+h)^2 = 2|x+h| - (x^2 + 2xh + h^2)$
 $= 2|x+h| - x^2 - 2xh - h^2$

2. Consider the system of equations

$$\begin{aligned} 3x - y &= 1 \\ x + \frac{1}{2}y &= -3 \end{aligned}$$

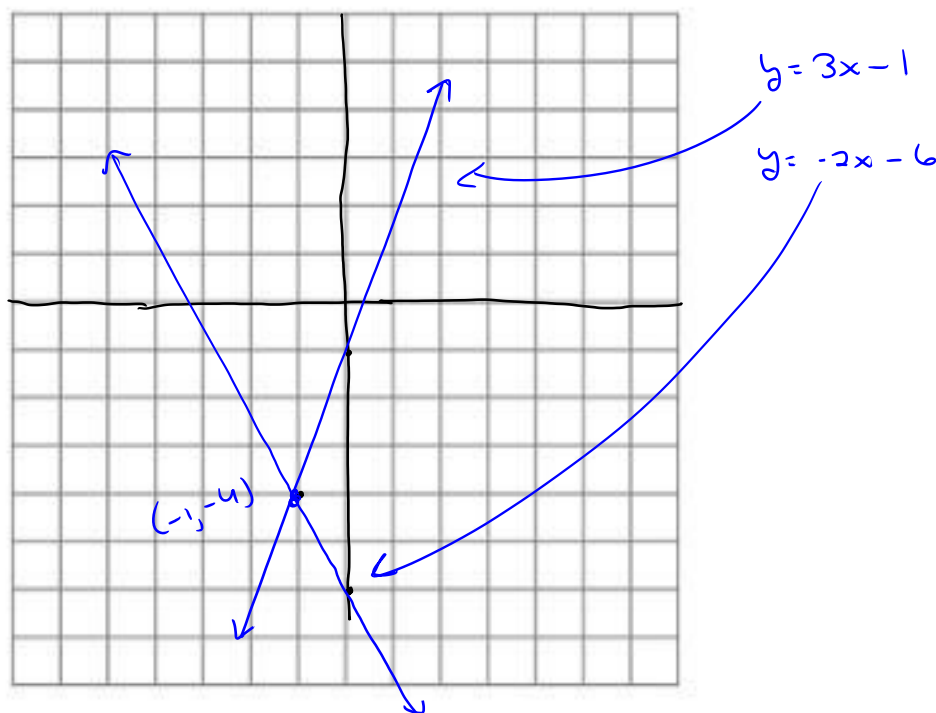
- (a) (5 points) Solve the linear system if possible, or show that it does not have any solutions.
(Use the technique of your choice.)

$$\begin{aligned} x &= -3 - \frac{1}{2}y & -\frac{5}{2}y &= 10 & x &= -3 - \frac{1}{2}(-4) = -3 + 2 = -1 \\ \text{Sub } 3(-3 - \frac{1}{2}y) - y &= 1 & y &= 10 \left(\frac{-2}{5}\right) & \text{The solution} & \\ & & &= -4 & \text{is} & \\ & & & & & (-1, -4) \end{aligned}$$

- (b) (3 points) State whether the system is consistent independent, consistent dependent, or inconsistent.

Consistent independent

- (c) (5 points) Plot the two lines defined by the equations in this system.



- (d) (2 points) State in your own words how the plot you created illustrates the solution, solutions, or the fact that the system is inconsistent (based on what you found it to be).

The lines intersect @ the one solution $(-1, -4)$.

3. For each circle, identify its center and its radius.

(a) (5 points) $(x + 2)^2 + (y - 7)^2 = 25$

Center $(-2, 7)$ radius 5

(b) (5 points) $x^2 + y^2 + 6x - 8y + 18 = 0$

$$x^2 + 6x + 9 + y^2 - 8y + 16 = -18 + 9 + 16$$

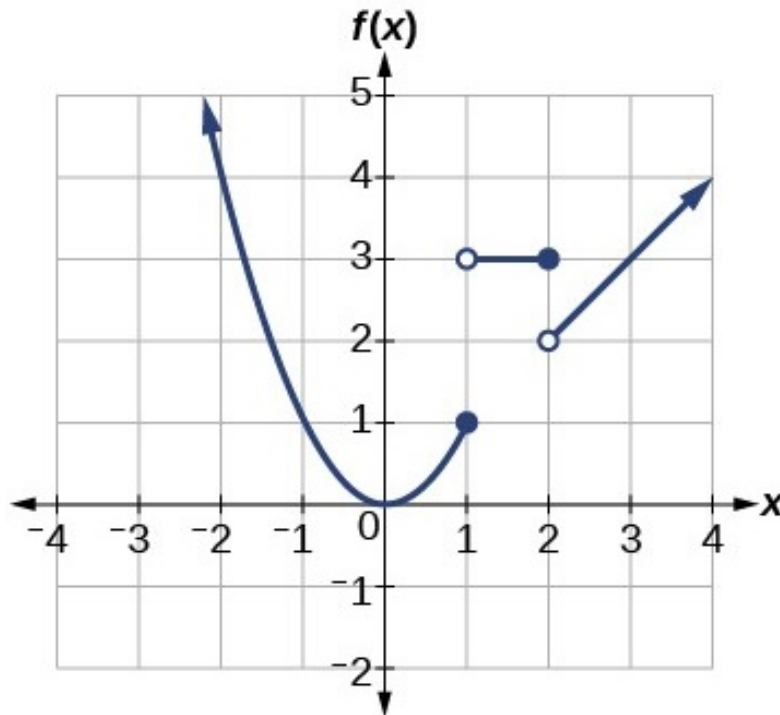
$$(x + 3)^2 + (y - 4)^2 = 7$$

Center $(-3, 4)$ radius $\sqrt{7}$

4. (10 points) Let $f(x) = 2x^2 - x$. Set up the difference quotient $\frac{f(x+h) - f(x)}{h}$, and simplify it as much as possible.

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{2(x+h)^2 - (x+h) - (2x^2 - x)}{h} \\ &= \frac{2(x^2 + 2xh + h^2) - x - h - 2x^2 + x}{h} \\ &= \frac{2x^2 + 4xh + 2h^2 - x - h - 2x^2 + x}{h} \\ &= \frac{4xh + 2h^2 - h}{h} = \frac{h(4x + 2h - 1)}{h} \\ &= 4x + 2h - 1 \end{aligned}$$

5. The figure shows the graph of $y = f(x)$. Use the graph to answer the following questions about the function f . (2 points each)



- (a) Evaluate $f(-2) = 4$
- (b) Evaluate $f(1) = 1$
- (c) Evaluate $f(2) = 3$
- (d) Evaluate $f(f(-2)) = f(4) = 4$
- (e) How many solutions are there to the equation $f(x) = 2$? *One*
- (f) Find all solutions to the equation $f(x) = 1$. *$x = -1$ and $x = 1$*
- (g) Is $-\frac{1}{2}$ in the domain of f ? *Yes*
- (h) Is $-\frac{1}{2}$ in the range of f ? *No*
- (i) On the interval $(1, 2)$, is f increasing, decreasing or constant? *It's constant*
- (j) Consider the statement " $f(x) \geq 0$ for all x in the domain of f ." Is this true?

Yes $f(x) \geq 0$. The entire graph is at or above the x-axis

6. Let

$$f(x) = 4x - 7 \quad \text{and} \quad g(x) = \frac{x}{x-1}.$$

Evaluate or answer the following questions about these functions. If a quantity is undefined, you can say that it is *undefined* or *doesn't exist*. (2 points each)

(a) What is the domain of f . Give your answer in interval notation. $(-\infty, \infty)$

(b) What is the domain of g . Give your answer in interval notation. $(-\infty, 1) \cup (1, \infty)$

(c) Evaluate $(f + g)(2) = f(2) + g(2) = 8 - 7 + \frac{2}{2-1} = 1 + 2 = 3$

(d) Evaluate $(g - f)(2) = g(2) - f(2) = 2 - 1 = 1$

(e) Evaluate $(fg)(3) = f(3)g(3) = (12 - 7)\left(\frac{3}{3-1}\right) = 5\left(\frac{3}{2}\right) = \frac{15}{2}$

(f) Evaluate $\left(\frac{g}{f}\right)(3) = \frac{\frac{3}{2}}{5} = \frac{3}{10}$

(g) Evaluate $(f \circ g)(2) = f(g(2)) = f\left(\frac{2}{2-1}\right) = f(2) = 1$

(h) Evaluate $(g \circ f)(2) = g(f(2)) = g(1)$ this doesn't exist

(i) Evaluate $(f \circ f)(a)$. Assume that a is any real number in the domain of f .

$$\begin{aligned}(f \circ f)(a) &= f(f(a)) = f(4a - 7) = 4(4a - 7) - 7 \\ &= 16a - 28 - 7 = 16a - 35\end{aligned}$$

(j) Evaluate $(g \circ g)(b)$. Assume that b is any real number in the domain of g .

$$\begin{aligned}(g \circ g)(b) &= g(g(b)) = g\left(\frac{b}{b-1}\right) \\ &= \frac{\frac{b}{b-1}}{\frac{b}{b-1} - 1} \cdot \left(\frac{b-1}{b-1}\right) = \frac{b}{b - (b-1)} = \frac{b}{1} = b\end{aligned}$$