Exam 1 Math 1190 sec. 51

Summer 2017

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ____________________

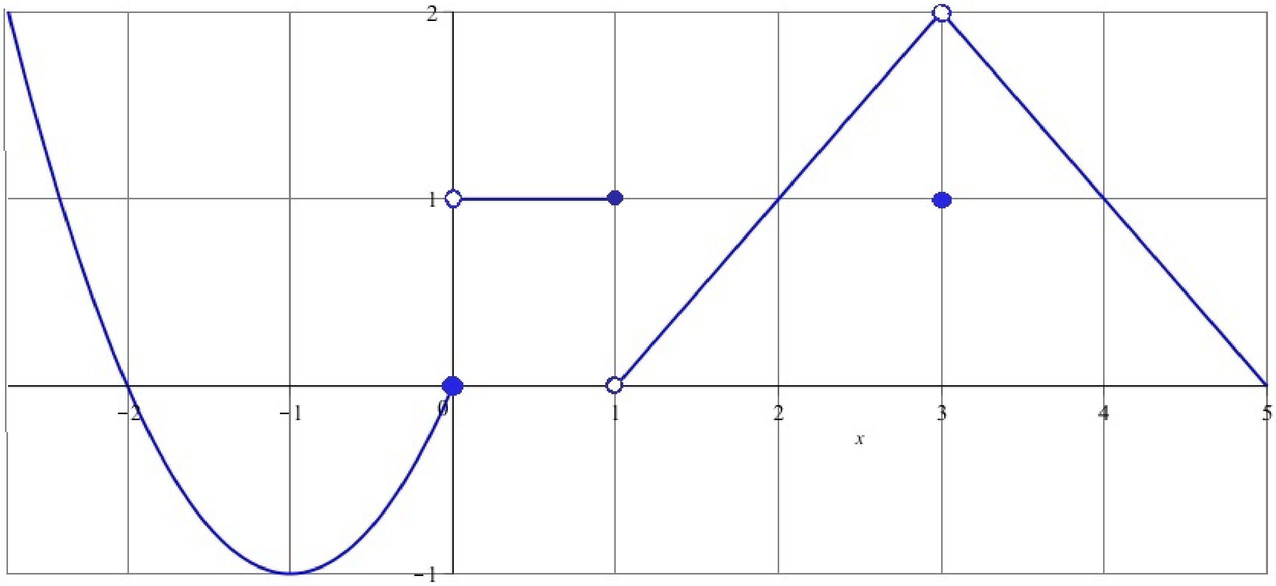
Problem	Points
1	
2	
3	
4	
5	
6	

INSTRUCTIONS: You have 60 minutes to complete this exam.

There are 6 problems. The point values are listed with the problems.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) (15 points) Use the graph of $y = f(x)$ shown to evaluate or answer the following questions.



a) (2 pts) Evaluate or write DNE $\lim_{x\to 0^+} f(x) =$ 1

b) (2 pts) Evaluate or write DNE $\lim_{x\to 0^-} f(x) =$ 0

c) (2 pts) Evaluate or write DNE $\lim_{x\to 1} f(x) =$ DNE

d) (2 pts) Evaluate or write DNE $f(3) =$ 1

e) (2 pts) Evaluate or write DNE $\lim_{x\to 3} f(x) =$ 2

f) (3 pts) List all x values on the interval $2 < x < 5$ at which f is discontinuous.

0, 1, and 3

g) (2 pts) Is f continuous **from the right**, continuous **from the left** or neither at $x = 0$? (Justify)

$f(0) = 0$ $\lim_{x\to 0^-} f(x) = 0 = f(0)$ ← it's continuous from the left

$\lim_{x\to 0^+} f(x) = 1 \neq f(0)$ ← not from the right.

(2) (20 points) Evaluate each limit using limits laws and any necessary algebra. (Use of l'Hospital's rule will not be considered for credit.)

(a) $\lim_{x\to 3} \frac{x^2+1}{x+2}$ $= \frac{3^2+1}{3+2} = \frac{10}{5} = 2$

(b) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$ $\cdot \left(\frac{\sqrt{x}+2}{\sqrt{x}+2}\right) = \lim_{x\to 4} \frac{(x-4)(\sqrt{x}+2)}{x-4}$

"$\frac{0}{0}$"

$= \lim_{x\to 4} (\sqrt{x}+2) = \sqrt{4}+2 = 4$

(c) $\lim_{x\to -2} \frac{x^2+5x+6}{x^2-4}$ $= \lim_{x\to -2} \frac{(x+2)(x+3)}{(x+2)(x-2)}$

"$\frac{0}{0}$"

$= \lim_{x\to -2} \frac{x+3}{x-2} = \frac{-2+3}{-2-2} = \frac{-1}{4}$

(d) $\lim_{x\to 0} e^{\cos x}$ $= e^{\cos 0} = e^{1} = e$

(3) (20 points) Evaluate each limit. If a limit is an infinity, ∞ or $-\infty$, give the appropriate infinity as your answer. (Use of l'Hospital's rule will not be considered for credit.)

(a) $\lim_{x\to\infty} \frac{8}{\sqrt[4]{x}}$ $= 0$

(b) $\lim_{x\to-\infty} \frac{2x-7}{3x+1}$ $= \lim_{x\to-\infty} \left(\frac{2x-7}{3x+1}\right)\cdot\frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to-\infty} \frac{2-\frac{7}{x}}{3+\frac{1}{x}} = \frac{2-0}{3+0} = \frac{2}{3}$

(c) $\lim_{x\to0^+} \ln(x)$ $= -\infty$

(d) $\lim_{x\to-1} \frac{x}{|x+1|}$

$\lim_{x\to-1} x = -1$, and $|x+1| > 0$ for all $x \neq -1$

But $\lim_{x\to-1} |x+1| = 0$

$= -\infty$

(4) Let $f(t) = \begin{cases} \dfrac{\sin(5t)+2t}{t}, & t \neq 0 \\ A, & t = 0 \end{cases}$ where A is some constant.

(a) (10 points) Evaluate $\lim_{t\to 0} f(t)$ $= \lim_{t\to 0} \dfrac{\sin(5t)+2t}{t}$

$$= \lim_{t\to 0} \left(\frac{\sin(5t)}{t} + \frac{2t}{t} \right)$$

$$= \lim_{t\to 0} \left(\frac{5}{5} \frac{\sin(5t)}{t} + 2 \right)$$

$$= \lim_{t\to 0} 5 \left(\frac{\sin(5t)}{t} \right) + \lim_{t\to 0} 2 = 5 \cdot 1 + 2 = 7$$

(b) (5 points) Determine what value A should take so that f will be continuous at $t = 0$.

If $A = 7$, then $\lim_{t\to 0} f(t) = f(0)$.

So A should be 7.

(5) (15 points) Suppose f and g are functions having the following known limits

$$\lim_{x\to -2} f(x) = -1, \quad \lim_{x\to 0} f(x) = 3, \quad \lim_{x\to 1} f(x) = 2, \quad \lim_{x\to 4} f(x) = -5,$$

$$\lim_{x\to -2} g(x) = 2, \quad \lim_{x\to 0} g(x) = -2, \quad \lim_{x\to 1} g(x) = 4, \quad \lim_{x\to 4} g(x) = 2.$$

Evaluate

a) $\lim_{x\to -2} 3f(x) =$ $3(-1) = -3$

b) $\lim_{x\to 1} (f(x) - 2g(x)) =$ $2 - 2 \cdot 4 = -6$

c) $\lim_{x\to 0} f(x)g(x) =$ $3(-2) = -6$

d) $\lim_{x\to 4} \frac{g(x)}{f(x)} =$ $\dfrac{2}{-5} = -\dfrac{2}{5}$

e) If f is continous at 4, evaluate $\lim_{x\to 1} f(g(x)) =$ $f\left(\lim_{x\to 1} g(x)\right) = f(4) = -5$

(6) Let $f(x) = x^2 + 3x$.

(a) (10 points) Evaluate $f'(2)$ by computing the limit (use proper notation!)

$$f'(2) = \lim_{x\to 2} \frac{f(x) - f(2)}{x-2} = \lim_{x\to 2} \frac{x^2+3x - (2^2+3\cdot 2)}{x-2}$$

$$= \lim_{x\to 2} \frac{x^2+3x-10}{x-2}$$

$$= \lim_{x\to 2} \frac{(x-2)(x+5)}{x-2}$$

$$= \lim_{x\to 2} (x+5) = 2+5 = 7$$

$$f'(2) = 7$$

(b) (5 points) Use your answer from part (a) to find the equation of the line tangent to the graph of f at the point $(2, f(2))$. Express your final answer in slope intercept form $y = mx + b$.

$f(2) = 10$ so the point is $(2, 10)$.

The slope $m_{tan} = f'(2) = 7$.

$$y - 10 = 7(x-2)$$

$$y - 10 = 7x - 14$$

$$y = 7x - 14 + 10$$

$$\boxed{y = 7x - 4}$$