(Part 2) Exam I Math 1190 sec. 51

Fall 2016

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

INSTRUCTIONS: There are 7 problems. The point values are listed with the problems; there are 65 possible points. This constitutes 65% of the first exam for this course.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.**

To receive full credit, answers must be clear, complete, justified, and written using proper notation.

Problem	Points
1	
2	
3	
4	
5	
6	
7	

(1) (10 points) Evaluate each limit using limit laws and any necessary algebra.

(a) $\lim_{x\to 2^+} \sqrt{x^2 - 4}$ $= \sqrt{2^2 - 4} = \sqrt{4-4} = 0$

(b) $\lim_{t\to 4} \frac{t-4}{\sqrt{t}-2}$ $= \lim_{t\to 4} \frac{t-4}{\sqrt{t}-2} \cdot \left(\frac{\sqrt{t}+2}{\sqrt{t}+2}\right)$

$= \lim_{t\to 4} \frac{(t-4)(\sqrt{t}+2)}{t-4} = \lim_{t\to 4} (\sqrt{t}+2)$

$= \sqrt{4}+2 = 4$

(2) (10 points) Use the known result $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ to evaluate the limit. Appropriate and correct steps must be shown to receive credit.

$\lim_{x\to 0} \frac{\tan(3x)}{4x}$ $= \lim_{x\to 0} \frac{1}{4} \frac{\frac{\sin(3x)}{\cos(3x)}}{x}$

$= \lim_{x\to 0} \frac{1}{4\cos(3x)} \frac{\sin(3x)}{x} \cdot \frac{3}{3}$

$= \lim_{x\to 0} \frac{3}{4\cos(3x)} \frac{\sin(3x)}{3x}$

$= \frac{3}{4\cdot 1} \cdot 1 = \frac{3}{4}$

(3) (a) (10 points) Let $f(x) = 3x^2 + 1$. Find the slope of the line tangent to the graph of f at the point $(1, f(1))$ by setting up and evaluating the limit

$$m_{tan} = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}.$$

$f(1) = 3 \cdot 1^2 + 1 = 4$

$$m_{tan} = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{(3x^2+1) - 4}{x - 1}$$

$$= \lim_{x \to 1} \frac{3x^2 - 3}{x - 1}$$

$$= \lim_{x \to 1} \frac{3(x-1)(x+1)}{x - 1}$$

$$= \lim_{x \to 1} 3(x+1) = 3(1+1) = 6$$

$$m_{tan} = 6$$

(b) (3 points) Use the results from part (a) to find the equation of the line tangent to the graph of f at the point $(1, f(1))$. Present your final answer in the slope-intercept form $y = mx + b$.

$$y - 4 = 6(x-1) \Rightarrow y = 6x - 6 + 4$$

$$\boxed{y = 6x - 2}$$

(4) (10 points) Evaluate each limit at infinity. If the limit is ∞ or $-\infty$ state this. If the limit doesn't exist, you may say DNE. Show work where appropriate to receive credit.

(a) $\lim_{x\to\infty} e^{2x}$ $= \infty$

(b) $\lim_{x\to\infty} \dfrac{x^4}{3x^4+2x^2+1}$ $= \lim_{x\to\infty} \dfrac{x^4}{3x^4+2x^2+1} \cdot \dfrac{\frac{1}{x^4}}{\frac{1}{x^4}}$

$= \lim_{x\to\infty} \dfrac{1}{3+\frac{2}{x^2}+\frac{1}{x^4}} = \dfrac{1}{3+0+0} = \dfrac{1}{3}$

(5) (10 points) Determine if the function is continuous at the indicated point. (Clearly state your conclusion with justification.)

$$f(x) = \begin{cases} e^{(x-1)}+1, & x<1 \\ 3x-1, & x \geq 1 \end{cases} \qquad c=1$$

$f(1) = 3\cdot 1 - 1 = 2$ $\quad$ $f(1)$ is defined

$\left.\begin{array}{l} \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} e^{x-1}+1 = e^0+1 = 2 \\ \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (3x-1) = 3-1 = 2 \end{array}\right\} \lim_{x\to 1} f(x) = 2$

Since $\lim_{x\to 1} f(x) = 2 = f(1)$

f is continuous @ 1.

(6) (6 points) Make a coherent argument that the equation $\sin x = 1 - x$ has at least one solution on the interval $[0, \pi]$.

Let $f(x) = \sin x - 1 + x$. Then f is continuous on $(-\infty, \infty)$. Hence f is continuous on $[0, \pi]$.

$f(0) = \sin(0) - 1 + 0 = -1 < 0$

$f(\pi) = \sin(\pi) - 1 + \pi = \pi - 1 > 0$ as $\pi > 1$

$N = 0$ is a number between $f(0)$ and $f(\pi)$.

By the IVT, there exists c in $(0, \pi)$ such that $f(c) = 0$. c is a solution since

$f(c) = \sin c - 1 + c = 0 \Rightarrow \sin c = 1 - c$.

(7) (6 points) Suppose f is continuous on $(-\infty, \infty)$ and $0 \leq f(x) \leq 2$ for all x. Determine $\lim_{x \to 0} x^2 f(x)$. (**Hint:** Since you don't know exactly what f is, see if you can use two other simpler functions to squeeze out an answer.)

$0 \leq f(x) \leq 2$ since $x^2 \geq 0$ for all x

$x^2 \cdot 0 \leq x^2 f(x) \leq x^2 \cdot 2 \Rightarrow 0 \leq x^2 f(x) \leq 2x^2$

This holds for all x. In particular for x near zero.

$\lim_{x \to 0} 0 = 0$, $\lim_{x \to 0} 2x^2 = 0$

By the squeeze thm

$\lim_{x \to 0} x^2 f(x) = 0$.