(Part 2) Exam I Math 1190 sec. 52

Fall 2016

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	

INSTRUCTIONS: There are 7 problems. The point values are listed with the problems; there are 65 possible points. This constitutes 65% of the first exam for this course.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.**

To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) (10 points) Evaluate each limit at infinity. If the limit is ∞ or $-\infty$ state this. If the limit doesn't exist, you may say DNE. Show work where appropriate to receive credit.

(a) $\lim_{x\to\infty} \frac{1-x}{2x+3} = \lim_{x\to\infty} \left(\frac{1-x}{2x+3}\right) \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{\frac{1}{x} - 1}{2 + 3/x} = \frac{0-1}{2+0} = \frac{-1}{2}$

(b) $\lim_{x\to\infty} \ln(x) = \infty$

(2) (10 points) Determine if the function is continuous at the indicated point. (Clearly state your conclusion with justification.)

$$f(x) = \begin{cases} \sin(\pi x) + 2, & x < 1 \\ 3x^2 - 1, & x \geq 1 \end{cases} \qquad c = 1$$

$f(1) = 3 \cdot 1^2 - 1 = 2$ $\qquad f(1)$ is defined

$\left.\begin{array}{l} \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (\sin(\pi x) + 2) = \sin(\pi) + 2 = 2 \\ \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (3x^2 - 1) = 3 \cdot 1^2 - 1 = 2 \end{array}\right\} \lim_{x\to 1} f(x) = 2$

$\boxed{\text{Since } \lim_{x\to 1} f(x) = 2 = f(1), \ f \text{ is continuous @ } 1.}$

(3) (10 points) Evaluate each limit using limit laws and any necessary algebra.

(a) $\lim_{x \to 3^+} \sqrt{x^2 - 9}$ $= \sqrt{3^2 - 9} = \sqrt{9 - 9} = 0$

(b) $\lim_{t \to 9} \frac{t - 9}{\sqrt{t} - 3}$ $= \lim_{t \to 9} \left(\frac{t - 9}{\sqrt{t} - 3} \right) \cdot \frac{\sqrt{t} + 3}{\sqrt{t} + 3}$

$= \lim_{t \to 9} \frac{(t - 9)(\sqrt{t} + 3)}{t - 9}$

$= \lim_{t \to 9} (\sqrt{t} + 3) = \sqrt{9} + 3 = 6$

(4) (10 points) Use the known result $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ to evaluate the limit. Appropriate and correct steps must be shown to receive credit.

$\lim_{x \to 0} x \csc(3x)$ $= \lim_{x \to 0} \frac{x}{\sin(3x)}$

$= \lim_{x \to 0} \frac{x}{\sin(3x)} \cdot \frac{3}{3}$

$= \lim_{x \to 0} \frac{1}{3} \frac{3x}{\sin(3x)}$

$= \frac{1}{3} \cdot \frac{1}{1} = \frac{1}{3}$

(5) (a) (10 points) Let $f(x) = 2x^2 + 2$. Find the slope of the line tangent to the graph of f at the point $(1, f(1))$ by setting up and evaluating the limit

$$m_{tan} = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}.$$

$f(1) = 2 \cdot 1^2 + 2 = 4$

$$m_{tan} = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{x \to 1} \frac{2x^2 + 2 - 4}{x - 1}$$

$$= \lim_{x \to 1} \frac{2x^2 - 2}{x - 1}$$

$$= \lim_{x \to 1} \frac{2(x-1)(x+1)}{x - 1}$$

$$= \lim_{x \to 1} 2(x+1) = 2(1+1) = 4$$

$$\boxed{m_{tan} = 4}$$

(b) (3 points) Use the results from part (a) to find the equation of the line tangent to the graph of f at the point $(1, f(1))$. Present your final answer in the slope-intercept form $y = mx + b$.

$y - 4 = 4(x - 1)$

$y = 4x - 4 + 4 \implies \boxed{y = 4x}$

(6) (6 points) Suppose f is continuous on $(-\infty, \infty)$ and $0 \le f(x) \le 3$ for all x. Determine $\lim_{x\to 0} x^2 f(x)$. (**Hint:** Since you don't know exactly what f is, see if you can use two other simpler functions to squeeze out an answer.)

$0 \le f(x) \le 3$ since $x^2 \geq 0$ for all x

$$0 \cdot x^2 \le x^2 f(x) \le 3 \cdot x^2$$

$$0 \le x^2 f(x) \le 3x^2 \quad \text{for all } x$$

This holds for x near zero.

$\lim_{x\to 0} 0 = 0$, $\lim_{x\to 0} 3x^2 = 0$

By the squeeze theorem $\lim_{x\to 0} x^2 f(x) = 0$.

(7) (6 points) Make a coherent argument that the equation $x = 1 - \sin x$ has at least one solution on the interval $[0, \pi]$.

Let $f(x) = x - 1 + \sin x$. f is continuous on $(-\infty, \infty)$. Hence f is continuous on $[0, \pi]$.

$f(0) = 0 - 1 + \sin 0 = -1 < 0$

$f(\pi) = \pi - 1 + \sin \pi = \pi - 1 > 0$ since $\pi > 1$

$N = 0$ is a number between $f(0)$ and $f(\pi)$.

By the IVT there exists c in $(0, \pi)$ such that $f(c) = 0$. This c solves the equation.

Note $f(c) = c - 1 + \sin c = 0 \Rightarrow c = 1 - \sin c$.