(Part 2) Exam I Math 1190 sec. 62

Spring 2017

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ____________________

Problem	Points
1	
2	
3	
4	
5	
6	

INSTRUCTIONS: You have 50 minutes to complete this exam.

There are 6 problems. The point values are listed with the problems; there are 65 possible points. This constitutes 65% of the first exam for this course.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) (15 points) Evaluate each limit using limit laws and any required algebra.

(a) $\lim_{x\to 1} \sqrt{3+x^2}$ $= \sqrt{3+1^2} = \sqrt{4} = 2$

(b) $\lim_{x\to -2} \dfrac{x^2-4}{x^2+5x+6}$ $=$ "$\frac{0}{0}$"

$$= \lim_{x\to -2} \frac{(x-2)\cancel{(x+2)}}{(x+3)\cancel{(x+2)}} = \lim_{x\to -2} \frac{x-2}{x+3} = \frac{-2-2}{-2+3} = \frac{-4}{1} = -4$$

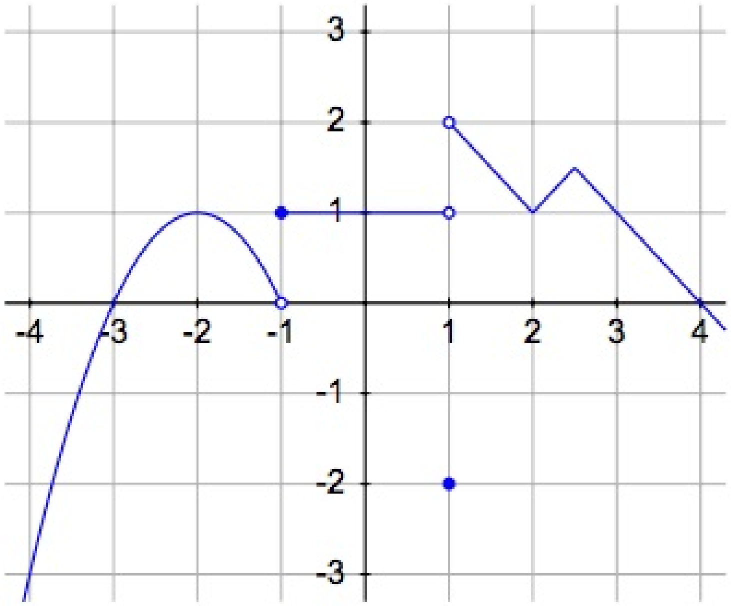
(c) $\lim_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}$ $=$ "$\frac{0}{0}$"

$$= \lim_{x\to 0} \left(\frac{\sqrt{x+1}-1}{x}\right)\left(\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right)$$

$$= \lim_{x\to 0} \frac{x+1-1}{x(\sqrt{x+1}+1)} = \lim_{x\to 0} \frac{x}{x(\sqrt{x+1}+1)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x+1}+1} = \frac{1}{\sqrt{0+1}+1} = \frac{1}{2}$$

(2) (10 points) Use the graph of $y = f(x)$ shown to evaluate or answer the following questions.



a Evaluate if possible $\lim_{x \to -3} f(x)$ = 0

b Evaluate if possible $f(1)$ = -2

c Evaluate if possible $\lim_{x \to -1^+} f(x)$ = 1

d Evaluate if possible $\lim_{x \to -1^-} f(x)$ = 0

e Determine whether f is continuous from the right at -1, continuous from the left at -1 or neither. (Justify your claim.)

Continuous from the right since

$\lim_{x \to 1^+} f(x) = 1 = f(-1)$

(3) (8 points) Use the identity $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ to evaluate the limit. (Show appropriate steps.)

$$\lim_{x\to 0} \frac{\sin(x)+\sin(3x)}{x} = \lim_{x\to 0} \left(\frac{\sin x}{x} + \frac{\sin(3x)}{x} \right)$$

$$= \lim_{x\to 0} \left(\frac{\sin x}{x} + \frac{\sin(3x)}{x} \cdot \frac{3}{3} \right)$$

$$= \lim_{x\to 0} \left(\frac{\sin x}{x} + 3 \frac{\sin(3x)}{3x} \right)$$

$$= 1 + 3 \cdot 1 = 4$$

(4) (10 points) Evaluate each limit if it exists. (Show steps where appropriate.)

(a) $\lim_{x\to\infty} \left(3 + \frac{3}{x}\right) = 3 + 0 = 3$

(b) $\lim_{x\to -\infty} \frac{2x^4+3x^2+1}{3x^4+x^3+x} \cdot \frac{\frac{1}{x^4}}{\frac{1}{x^4}}$

$$= \lim_{x\to -\infty} \frac{2 + \frac{3}{x^2} + \frac{1}{x^4}}{3 + \frac{1}{x} + \frac{1}{x^3}} = \frac{2+0+0}{3+0+0} = \frac{2}{3}$$

(5) (12 points) Suppose $f(x) = \begin{cases} Ax - 2, & x \leq 1 \\ 4x + 2, & x > 1 \end{cases}$ for some real number A.

(a) Evaluate $\lim_{x \to 1^-} f(x)$ $= \lim_{x \to 1^-} (Ax - 2) = A - 2$

(b) Evaluate $\lim_{x \to 1^+} f(x)$ $= \lim_{x \to 1^+} (4x + 2) = 4 \cdot 1 + 2 = 6$

(c) Determine all possible values of A for which f is continuous at 1.

we require $\lim_{x \to 1} f(x) = f(1) \Rightarrow A - 2 = 6 \Rightarrow A = 8$

(6) (10 points) Evaluate each limit. (These you should know. It is not necessary to try to show *work*).

a $\lim_{x \to \infty} e^x$ $= \infty$

b $\lim_{x \to -\infty} e^x$ $= 0$

c $\lim_{x \to \infty} \ln x$ $= \infty$

d $\lim_{x \to 0^+} \ln x$ $= -\infty$

e $\lim_{x \to 0} \cos x$ $= \cos 0 = 1$