(Part 2) Exam I Math 1190 sec. 63

Spring 2017

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ____________________

Problem	Points
1	
2	
3	
4	
5	
6	

INSTRUCTIONS: You have 50 minutes to complete this exam.

There are 6 problems. The point values are listed with the problems; there are 65 possible points. This constitutes 65% of the first exam for this course.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) (15 points) Evaluate each limit using limit laws and any required algebra.

(a) $\lim_{x\to 1} \frac{3}{\sqrt{4x+5}}$ $= \frac{3}{\sqrt{4\cdot 1+5}} = \frac{3}{\sqrt{9}} = \frac{3}{3} = 1$

(b) $\lim_{x\to -4} \frac{x^2-16}{x^2+7x+12}$ $= \text{"}\frac{0}{0}\text{"}$

$= \lim_{x\to -4} \frac{(x-4)\cancel{(x+4)}}{(x+3)\cancel{(x+4)}} =$

$= \lim_{x\to -4} \frac{x-4}{x+3} = \frac{-4-4}{-4+3} = \frac{-8}{-1} = 8$

(c) $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$ $= \text{"}\frac{0}{0}\text{"}$

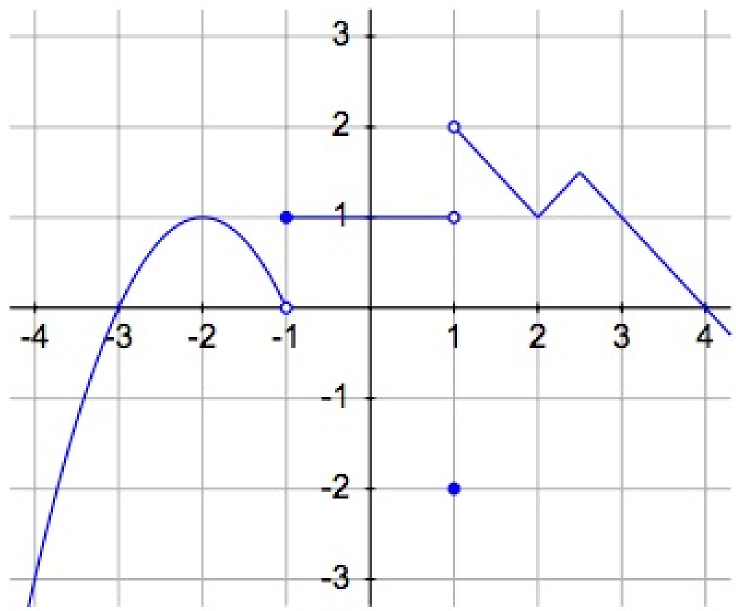
$= \lim_{x\to 0} \left(\frac{\sqrt{x+4}-2}{x}\right)\left(\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}\right)$

$= \lim_{x\to 0} \frac{x+4-4}{x(\sqrt{x+4}+2)} = \lim_{x\to 0} \frac{x}{x(\sqrt{x+4}+2)}$

$= \lim_{x\to 0} \frac{1}{\sqrt{x+4}+2} = \frac{1}{\sqrt{0+4}+2}$

$= \frac{1}{2+2} = \frac{1}{4}$

(2) (10 points) Use the graph of $y = f(x)$ shown to evaluate or answer the following questions.



a Evaluate if possible $\lim_{x \to -2} f(x)$ $= 1$

b Evaluate if possible $f(-1)$ $= 1$

c Evaluate if possible $\lim_{x \to 1^+} f(x)$ $= 2$

d Evaluate if possible $\lim_{x \to 1^-} f(x)$ $= 1$

e Determine whether f is continuous from the right at 1, continuous from the left at 1 or neither. (Justify your claim.)

Neither. $f(1) = -2$

$\lim_{x \to 1^-} f(x) = 1 \neq -2$ and $\lim_{x \to 1^+} f(x) = 2 \neq -2$

(3) (8 points) Use the identity $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ to evaluate the limit. (Show appropriate steps.)

$$\lim_{x \to 0} \frac{\sin(5x) + 2x}{x} = \lim_{x \to 0} \left(\frac{\sin(5x)}{x} + \frac{2x}{x} \right)$$

$$= \lim_{x \to 0} \left(\frac{\sin(5x)}{x} \cdot \frac{5}{5} + 2 \right)$$

$$= \lim_{x \to 0} \left(5 \left(\frac{\sin(5x)}{5x} \right) + 2 \right)$$

$$= 5 \cdot 1 + 2 = 7$$

(4) (10 points) Evaluate each limit if it exists. (Show steps where appropriate.)

(a) $\lim_{x \to \infty} \left(\frac{2}{x^2} - 4 \right) = 0 - 4 = -4$

(b) $$\lim_{x \to -\infty} \frac{5x^3 + 2x^2 + x + 2}{10x^3 + x - 4} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x \to -\infty} \frac{5 + \frac{2}{x} + \frac{1}{x^2} + \frac{2}{x^3}}{10 + \frac{1}{x^2} - \frac{4}{x^3}}$$

$$= \frac{5 + 0 + 0 + 0}{10 + 0 - 0} = \frac{5}{10} = \frac{1}{2}$$

(5) (12 points) Suppose $f(x) = \begin{cases} 3x + A, & x < 2 \\ x^2 + 1, & x \geq 2 \end{cases}$ for some real number A.

(a) Evaluate $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (3x + A) = 3 \cdot 2 + A = 6 + A$

(b) Evaluate $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^2 + 1) = 2^2 + 1 = 5$

(c) Determine all possible values of A for which f is continuous at 2.

we need $\lim_{x \to 2} f(x) = f(2) \Rightarrow 6 + A = 5$

$A = -1$

(6) (10 points) Evaluate each limit. (These you should know. It is not necessary to try to show *work*).

a $\lim_{x \to \infty} \ln x = \infty$

b $\lim_{x \to 0^+} \ln x = -\infty$

c $\lim_{x \to \infty} e^x = \infty$

d $\lim_{x \to -\infty} e^x = 0$

e $\lim_{x \to \frac{\pi}{2}} \sin x = \sin \pi/2 = 1$