Exam 1 Math 1112 sec. 52 Spring 2020

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ____________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

INSTRUCTIONS: There are 10 problems (some with multiple parts), worth 10 points each. There are no notes or books, allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, and written using proper notation.

No Calculator Use Is Allowed on These Pages

When you finish this part of the exam, turn it in to receive the additional questions for which calculator use is allowed.

1. (a) Find the equation of the line that is **parallel** to the line $4x - 2y = 3$ and passes through the point $(-2, 4)$.

Given line: $4x - 3 = 2y \Rightarrow y = 2x - 3/2$

$m = 2$ $\quad y - 4 = 2(x+2) = 2x+4$

$$\boxed{y = 2x + 8}$$

(b) Find the slope of any line that is **perpendicular** to the line $4x - 2y = 3$.

The given line has slope 2, so any perpendicular line will have slope $m = \frac{-1}{2}$.

2. Evaluate each expression exactly.

(a) $\log_5(5) =$ 1

(b) $\log_3\left(\frac{1}{27}\right)$ -3

(c) $\ln\left(\sqrt{e}\right) =$ $\frac{1}{2}$

(d) $\log(0.01) =$ -2

(e) $\log_{\frac{1}{2}}(1) =$ 0

3. Solve the exponential equation for x. Give an exact answer in terms of either the base-10 or the base-e logarithm.

$2^{x+1} = 9^{x-1}$

$\ln 2^{x+1} = \ln 9^{x-1}$

$(x+1)\ln 2 = (x-1)\ln 9$

$x\ln 2 + \ln 2 = x\ln 9 - \ln 9$

$\ln 2 + \ln 9 = x\ln 9 - x\ln 2 = x(\ln 9 - \ln 2)$

$$\boxed{x = \frac{\ln 2 + \ln 9}{\ln 9 - \ln 2}}$$

4. Write the expression as a single logarithm.

$4\log_4(x) - 4\log_4(y) + \frac{1}{3}\log_4(z) + \log_4(w)$

$= \log_4 x^4 - \log_4 y^4 + \log_4 z^{1/3} + \log_4 w$

$= \log_4\left(\frac{x^4 z^{1/3} w}{y^4}\right)$

$= \log_4\left(\frac{x^4 w \sqrt[3]{z}}{y^4}\right)$

5. Solve the logarithmic equation for x. (Hint: $1 = \log_4(4)$.)

$$\log_4(x-2) = 1 - \log_4(x-5)$$

$\log_4(x-2) + \log_4(x-5) = 1$

$\log_4((x-2)(x-5)) = \log_4 4 \Rightarrow \log_4(x^2 - 7x + 10) = \log_4 4$

$x^2 - 7x + 10 = 4 \Rightarrow x^2 - 7x + 6 = 0$

$(x-1)(x-6) = 0$

$x = 1$ or $x = 6$.

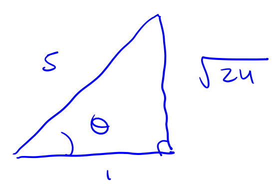
We require $x - 2 > 0$ and $x - 5 > 0$ so $x > 5$.

Hence 1 is not a solution.

There is one solution $x = 6$

6. Suppose θ is an acute angle and $\cos\theta = \dfrac{1}{5}$. Find the remaining five trigonometric values of θ. Record your answers in the spaces provided. (Your answers should be exact, not decimal approximations.)

Representative triangle



$1^2 + a^2 = 5^2 \Rightarrow a^2 = 25 - 1 = 24$

$a = \sqrt{24}$

$\sin\theta = \dfrac{\sqrt{24}}{5}$ $\tan\theta = \sqrt{24}$ $\csc\theta = \dfrac{5}{\sqrt{24}}$ $\cot\theta = \dfrac{1}{\sqrt{24}}$ $\sec\theta = 5$

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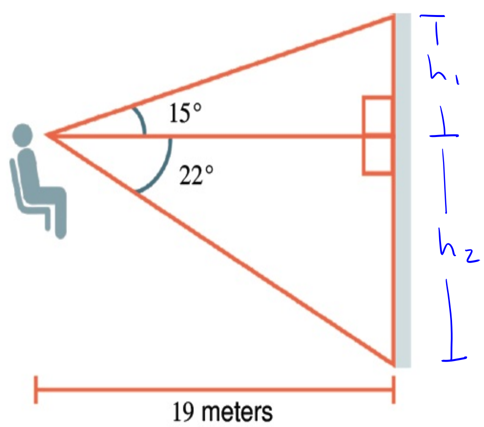
You may use a calculator on these questions.

7. Jenny is sitting in a movie theater, 19 meters from the screen. The angle of elevation from her line of sight to the top of the screen is 15°, and the angle of depression from her line of sight to the bottom of the screen is 22°.

Find the height of the entire screen. Do not round any intermediate computations. Round your answer to the nearest tenth.



Let h_1 and h_2 be the segments labeled.

The total height

$$h = h_1 + h_2$$

$$\frac{h_1}{19\,m} = \tan 15^\circ \quad \text{and} \quad \frac{h_2}{19\,m} = \tan 22^\circ$$

$$h_1 = 19 \tan 15^\circ \, m \quad \text{and} \quad h_2 = 19 \tan 22^\circ \, m$$

The height h of the screen

$$h = 19 \tan 15^\circ + 19 \tan 22^\circ \, m$$

$$\approx 12.8 \text{ meters}$$

8. A ceiling fan has blades that are 18 inches (so the radius of the circular fan is 18 inches). Suppose the linear speed of the tip of the blade is 6 feet per second.

(a) Find the angular speed of the fan in radians per minute.

(b) Find the number of revolutions a blade makes per minute. (To the nearest whole number.)

Some facts given below may or may not be useful.

The linear speed $\nu = 6 \frac{ft}{sec}$.

From $\nu = r\omega$, and $r = 18$ in $= \frac{3}{2}$ ft

a) $$\omega = \frac{\nu}{r} = \frac{6 \frac{ft}{sec}}{3/2 \text{ ft}} = 4 \frac{rad}{sec} \cdot 60 \frac{sec}{min}$$

$$\omega = 240 \frac{rad}{min}$$

b) Each rotation is 2π radians, so in revolutions per minute

$$\omega = \frac{240}{2\pi} \frac{rev}{min} \approx 38 \frac{rev}{min}$$

- There are 60 seconds in one minute.
- The fan blades are oak.
- Linear speed ν and angular speed ω are related by $\nu = r\omega$.
- There are 2π radians in one revolution.
- The ceiling fan is rotating counter clock-wise when viewed from below.

9. Solve each equation for x. Do not round any intermediate computations, and round your final answer to the nearest thousandth.

(a) $6 + 4\ln(2x) = 9$ $\Rightarrow$ $4\ln(2x) = 3$ $\Rightarrow$ $\ln(2x) = \frac{3}{4}$

$2x = e^{3/4}$ $\Rightarrow$ $x = \frac{1}{2}e^{3/4}$

≈ 1.059

(b) $\dfrac{8}{1+2e^{-x}} = 4$ $\Rightarrow$ $4(1+2e^{-x}) = 8$ $\Rightarrow$ $1 + 2e^{-x} = 2$

$2e^{-x} = 1$ $\Rightarrow$ $e^{-x} = \frac{1}{2}$

$-x = \ln(\frac{1}{2})$

$x = -\ln(\frac{1}{2}) \approx 0.693$

10. Consider the function $P(n) = 0.5n + 6.4$, where P is the price in dollars for n grams of vitamins. Let P^{-1} be the inverse function, and x be an output of P.

n - grams, $P \sim \$$ so x - $\$$ $P^{-1} \sim$ grams

(a) Which of the following best describes $P^{-1}(x)$? (circle one)

(i) The price (in dollars) for x grams of vitamins.

(ii) The amount of vitamins (in grams) for a price of x dollars.

(iii) The ratio of the price (in dollars) to the number of grams x.

(iv) The reciprocal of the price (in dollars) for x grams of vitamins.

(b) Find $P^{-1}(9.4)$.

$9.4 = 0.5n + 6.4$ $\Rightarrow$ $0.5n = 3$ $n = 3/0.5 = 6$

$P^{-1}(9.4) = 6$ (those are grams)