Exam 2 Math 1113 sec. 51 Fall 2018

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: _____

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

INSTRUCTIONS: There are 10 problems worth 10 points each. You may use a calculator that does not have symbolic manipulation features, but you do not need a calculator. There are no notes, or books allowed. **Illicit use of a smart phone, tablet, device that runs apps, or notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, and written using proper notation.

(1) Find all zeros of the polynomial function, and state the multiplicity of each zero. (**NOTE:** These are only partially factored. Complete the factorization to correctly identify zeros and their multiplicities.)

(a) $f(x) = (x^2-1)^2(x+2)^3$ $= (x-1)^2(x+1)^2(x+2)^3$

zeros	mult.
1	2
-1	2
-2	3

(b) $g(x) = (x+3)(x-4)(x^2-16)$ $= (x+3)(x-4)^2(x+4)$

zeros	mult.
-3	1
4	2
-4	1

(2) Simplify the complex rational expression.

$$\frac{\frac{2}{x+1}-\frac{1}{2}}{x-3}$$

$\cdot \left(\frac{2(x+1)}{2(x+1)}\right) = \frac{4-(x+1)}{(x-3)2(x+1)} = \frac{3-x}{(x-3)2(x+1)}$

$= \frac{-(x-3)}{(x-3)2(x+1)} = \frac{-1}{2(x+1)}$

(3) Write each quadratic function in vertex form. Identify the vertex and the equation of the axis of symmetry.

(a) $p(x) = x^2-8x+18$ $= (x^2-8x+16-16)+18$

$= (x-4)^2+2$

vertex $(4,2)$ axis of symmetry $x=4$

(b) $q(x) = 2x^2+12x+13$ $= 2(x^2+6x+9-9)+13 = 2(x+3)^2-5$

vertex $(-3,-5)$ axis $x=-3$

(4) Find all solutions of the polynomial equation. (Hint: Factor completely.)

$$3(x+3)^2(x-2)^5+5(x+3)^3(x-2)^4=0$$

$(x+3)^2(x-2)^4\left(3(x-2)+5(x+3)\right)=0$

$(x+3)^2(x-2)^4(8x+9)=0$

$x=-3$, $x=2$ or $x=\frac{-9}{8}$

(5) A company can produce up to 100 widgets a day. If x widgets are produced, the day's revenue R and cost C are known to be

$$R(x) = x(100 - x) \quad \text{and} \quad C(x) = 10x + 500.$$

(a) Determine the quadratic function $P(x) = (R - C)(x)$ representing the day's profit. Write your answer in standard form (that is, in the form $ax^2 + bx + c$).

$P(x) = 100x - x^2 - (10x + 500) = -x^2 + 90x - 500$

$P(x) = -x^2 + 90x - 500$

(b) Determine the number of widgets that should be produced to maximize the daily profit.

The maximum happens @ the vertex where $x = \frac{-b}{2a}$

The best x value is

$x = \frac{-90}{2(-1)} = 45$ 45 widgets should be made.

(6) For the rational function $f(x) = \dfrac{2x^2 + 5x - 3}{x^2 - 9} = \dfrac{(2x-1)(x+3)}{(x-3)(x+3)} = \dfrac{2x-1}{x-3}$, $x \neq -3$

(a) Identify the domain. (Use interval or set builder notation, your choice.)

$\{x \mid x \neq \pm 3\}$ a.k.a $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$.

(b) Determine the equation(s) of all vertical asymptote(s).

$x = 3$ (there's a hole @ -3)

(c) Determine if the graph of f has a horizontal asymptote. If so, determine its equation.

$n = m = 2$ There is one. $a_n = 2$ $b_m = 1$

$y = 2$

(7) Use long division to find the equation of the oblique asymptote to the graph of the rational function, and determine whether the graph of f crosses the oblique asymptote.

$$f(x) = \frac{2x^2 + 9x}{x + 5}$$

$f(x) = 2x - 1 + \frac{5}{x+5}$

$$\begin{array}{r} 2x-1 \\ x+5 \overline{)\,2x^2+9x+0} \\ -(2x^2+10x) \\ \hline -x+0 \\ -(-x-5) \\ \hline 5 \end{array}$$

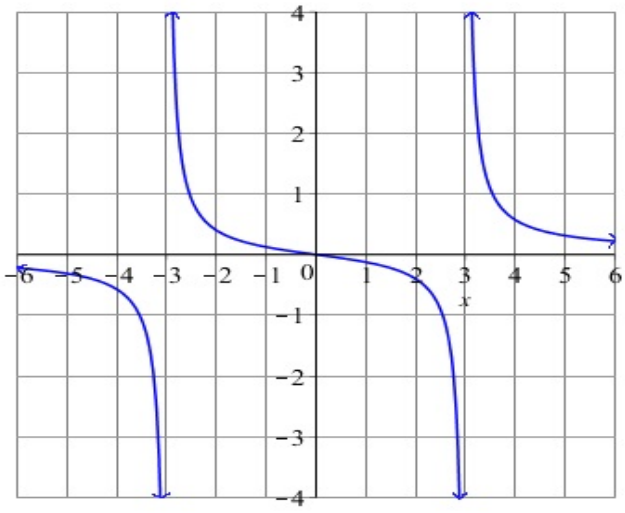
The asymptote is $y = 2x - 1$

$2x - 1 + \frac{5}{x+5} = 2x - 1 \Rightarrow \frac{5}{x+5} = 0$

$\Rightarrow 5 = 0$ false

It doesn't cross.

(8) The figure shows the graph of the rational function $f(x) = \dfrac{x}{x^2 - 9}$. Refering to the figure and formula for f given, determine which of the following statements are true and which are false. (Indicate with T or F).



(a) $f(x) \to \infty$ as $x \to 3^-$ F

(b) $f(x) \to \infty$ as $x \to -3^+$ T

(c) f is an odd function. T

(d) $f(x) \to -\infty$ as $x \to 3$ F

(e) f has a hole in its graph at $(0, 0)$. F

(9) For each polynomial function, identify

(i) the degree,

(ii) the leading term,

(iii) the maximum possible number of real zeros,

(iv) the maximum possible number of turning points, and

(v) the nature of the end behavior.

(a) $p(x) = 7-14x+3x^2-4x^3+12x^5$

i) 5

ii) $12x^5$

iii) 5

iv) 4

v) du (last one)

(b) $q(x) = -3x^6-12x^5+30x^4+120x^3-135x^2-324x+324$

i) 6

ii) $-3x^6$

iii) 6

iv) 5

v) dd (first one)

(10) Simplify each expression. Assume that variables can represent any real number.

(a) $\sqrt[3]{8x^3}$ $= 2x$

(b) $\sqrt{(x+3)^2}$ $= |x+3|$

(c) $\sqrt[4]{\dfrac{16}{x^8}}$ $= \dfrac{2}{x^2}$

(d) $\sqrt{(-y)^2}$ $= |y|$

(e) $\sqrt{2x^2}\sqrt{8x^2}$ $= 4x^2$