

# Exam 2 Math 1113 sec. 52 Fall 2018

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: \_\_\_\_\_

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

INSTRUCTIONS: There are 10 problems worth 10 points each. You may use a calculator that does not have symbolic manipulation features, but you do not need a calculator. There are no notes, or books allowed. **Illicit use of a smart phone, tablet, device that runs apps, or notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, and written using proper notation.

(1) Find all zeros of the polynomial function, and state the multiplicity of each zero. (NOTE: These are only partially factored. Complete the factorization to correctly identify zeros and their multiplicities.)

(a)  $g(x) = (x+3)(x-4)(x^2-9) = (x+3)^2(x-3)(x-4)$

Zeros	multiplicity
-3	2
3	1
4	1

(b)  $f(x) = (x^2-1)^2(x+5)^3 = (x-1)^2(x+1)^2(x+5)^3$

zero	mults.
1	2
-1	2
-5	3

(2) Simplify the complex rational expression.

$$\frac{\frac{1}{x+2} - \frac{1}{x-2}}{\frac{4(x+2)}{4(x+2)}} = \frac{4 - (x+2)}{(x-2)4(x+2)} = \frac{2-x}{(x-2)4(x+2)}$$
$$= \frac{-(x-2)}{(x-2)4(x+2)} = \frac{-1}{4(x+2)}$$

(3) Write each quadratic function in vertex form. Identify the vertex and the equation of the axis of symmetry.

(a)  $p(x) = 2x^2 - 12x + 14 = 2(x^2 - 6x + 9 - 9) + 14$   
 $= 2(x-3)^2 - 4$   
vertex  $(3, -4)$  axis of symmetry  $x = 3$

(b)  $q(x) = x^2 + 10x + 30 = (x^2 + 10x + 25 - 25) + 30 = (x+5)^2 + 5$   
vertex  $(-5, 5)$  axis  $x = -5$

(4) Find all solutions of the polynomial equation. (Hint: Factor completely.)

$$4(x-3)^3(x+2)^2 + 2(x-3)^4(x+2) = 0$$
$$(x-3)^3(x+2) (4(x+2) + 2(x-3)) = 0$$
$$(x-3)^3(x+2) (6x+2) = 0$$
$$x = 3, \quad x = -2, \quad \text{or} \quad x = \frac{-2}{6} = \frac{-1}{3}$$

(5) A company can produce up to 100 widgets a day. If  $x$  widgets are produced, the day's revenue  $R$  and cost  $C$  are known to be

$$R(x) = x(100 - x) \quad \text{and} \quad C(x) = 10x + 500.$$

- (a) Determine the quadratic function  $P(x) = (R - C)(x)$  representing the day's profit. Write your answer in standard form (that is, in the form  $ax^2 + bx + c$ ).

$$P(x) = 100x - x^2 - (10x + 500) = -x^2 + 90x - 500$$

$$P(x) = -x^2 + 90x - 500$$

- (b) Determine the number of widgets that should be produced to maximize the daily profit.

The maximum is @ the vertex where  $x = \frac{-b}{2a}$ .

The best widget production is

$$x = \frac{-90}{2(-1)} = 45 \text{ units}$$

- (6) For the rational function  $f(x) = \frac{2x^2 + 7x - 4}{x^2 - 16} = \frac{(2x-1)(x+4)}{(x-4)(x+4)} = \frac{2x-1}{x-4}$ ,  $x \neq -4$

- (a) Identify the domain. (Use interval or set builder notation, your choice.)

$$\{x \mid x \neq \pm 4\} \quad \text{i.e.} \quad (-\infty, -4) \cup (-4, 4) \cup (4, \infty)$$

- (b) Determine the equation(s) of all vertical asymptote(s).

$$x = 4 \quad (\text{there is a hole @ } -4)$$

- (c) Determine if the graph of  $f$  has a horizontal asymptote. If so, determine its equation.

$$n=m=2 \text{ so yes there is one. } a_n=2, b_n=1$$

$$\underline{\underline{y = 2}}$$

(7) Use long division to find the equation of the oblique asymptote to the graph of the rational function, and determine whether the graph of  $f$  crosses the oblique asymptote.

$$f(x) = \frac{3x^2 + 8x + 7}{x + 2}$$

$$= 3x + 2 + \frac{3}{x + 2}$$

The asymptote is  
 $y = 3x + 2$

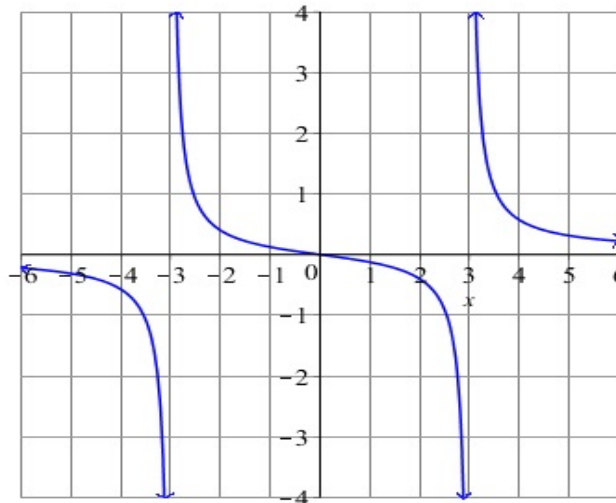
$$\begin{array}{r} 3x + 2 \\ x + 2 \overline{) 3x^2 + 8x + 7} \\ \underline{-(3x^2 + 6x)} \phantom{+ 7} \\ 2x + 7 \\ \underline{-(2x + 4)} \\ 3 \end{array}$$

$$f(x) = 3x + 2 \Rightarrow \frac{3}{x + 2} = 0$$

$3 = 0$  false

It doesn't cross

(8) The figure shows the graph of the rational function  $f(x) = \frac{x}{x^2 - 9}$ . Referring to the figure and formula for  $f$  given, determine which of the following statements are true and which are false. (Indicate with T or F).



- (a)  $f(x) \rightarrow \infty$  as  $x \rightarrow 3^+$       T
- (b)  $f(x) \rightarrow \infty$  as  $x \rightarrow -3^-$       F
- (c)  $f$  is an odd function.      T
- (d)  $f(x) \rightarrow -\infty$  as  $x \rightarrow 3$       F
- (e)  $f$  has a hole in its graph at  $(0, 0)$ .      F

(9) For each polynomial function, identify

- (i) the degree,
- (ii) the leading term,
- (iii) the maximum possible number of real zeros,
- (iv) the maximum possible number of turning points, and
- (v) the nature of the end behavior.



(a)  $p(x) = 7 - 14x + 3x^2 - 4x^3 + 12x^5$

- i) 5
- ii)  $12x^5$
- iii) 5
- iv) 4
- v) du



(b)  $q(x) = -3x^6 - 12x^5 + 30x^4 + 120x^3 - 135x^2 - 324x + 324$

- i) 6
- ii)  $-3x^6$
- iii) 6
- iv) 5
- v) dd ↓ ↓

(10) Simplify each expression. Assume that variables can represent any real number.

(a)  $\sqrt[4]{\frac{16}{x^8}} = \frac{2}{x^2}$

(b)  $\sqrt{2x^2}\sqrt{8x^2} = 4x^2$

(c)  $\sqrt{(x+3)^2} = |x+3|$

(d)  $\sqrt[3]{8x^3} = 2x$

(e)  $\sqrt{(-y)^2} = |y|$