Exam 2 Math 1190 sec. 51

Fall 2016

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

INSTRUCTIONS: There are 7 problems. The point values are listed with the problems.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.**

To receive full credit, answers must be clear, complete, justified, and written using proper notation.

Problem	Points
1	
2	
3	
4	
5	
6	
7	

(1) (15 points, 5 each) Find the derivative of each function. Do not leave compound fractions in your answers, otherwise it is not necessary to simplify.

(a) $y = x^2 \cos(x)$

$y' = 2x \cos x - x^2 \sin x$

(b) $f(x) = \dfrac{\tan x}{x^3 + x}$

$f'(x) = \dfrac{\sec^2 x \, (x^3+x) - \tan x \, (3x^2+1)}{(x^3+x)^2}$

(c) $y = \sqrt[3]{x^4} = x^{4/3}$

$y' = \frac{4}{3} x^{\frac{1}{3}} = \frac{4}{3} \sqrt[3]{x}$

(2) (15 points) Find $\frac{dy}{dx}$ given the relation $x^2y = 2x - y^3$.

$$2xy + x^2\frac{dy}{dx} = 2 - 3y^2\frac{dy}{dx}$$

$$x^2\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 2 - 2xy$$

$$(x^2 + 3y^2)\frac{dy}{dx} = 2 - 2xy$$

$$\boxed{\frac{dy}{dx} = \frac{2-2xy}{x^2+3y^2}}$$

(3) (15 points) Find the equation of the line tangent to the graph of $f(x) = x^2e^x$ at the point $(1, e)$.

$$f'(x) = 2xe^x + x^2e^x \quad , \quad f'(1) = 2\cdot 1 e^1 + 1^2 e^1 = 3e$$

$$y - e = 3e(x-1) = 3ex - 3e$$

$$y = 3ex - 3e + e$$

$$\boxed{y = 3ex - 2e}$$

(4) (15 points, 5 each) Find the derivative of each function.

(a) $f(x) = \sin^{-1}(x^2)$

$$f'(x) = \frac{1}{\sqrt{1-x^4}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$$

(b) $f(x) = x\tan^{-1} x$

$$f'(x) = \tan^{-1} x + x \frac{1}{1+x^2}$$

$$= \tan^{-1} x + \frac{x}{1+x^2}$$

(c) $g(t) = 3^t + \log_3(t)$

$$g'(t) = 3^t \ln 3 + \frac{1}{t \ln 3}$$

(5) (10 points) Find the x-value of all points on the graph of the function at which the tangent line is horizontal.

$f(x) = (x-2)^3(x+5)^{-4}$

$$f'(x) = 3(x-2)^2(x+5)^{-4} - 4(x-2)^3(x+5)^{-5}$$

$$3(x-2)^2(x+5)^{-4} - 4(x-2)^3(x+5)^{-5} = 0$$

$$(x-2)^2(x+5)^{-5}[3(x+5) - 4(x-2)] = 0$$

$$(x-2)^2(x+5)^{-5}(-x+23) = 0 \Rightarrow x = 2 \text{ or } x = 23$$

There are horizontal tangents @ $x=2$ and @ $x=23$.

(6) (10 points) Use logarithmic differentiation to find $\frac{dy}{dx}$. Assume y is positive. (Your final result should be stated in terms of x, but need not be fully simplified.)

$y = (\cot x)^x$

$$\ln y = \ln(\cot x)^x = x \ln \cot x$$

$$\frac{1}{y}\frac{dy}{dx} = \ln \cot x + x \frac{-\csc^2 x}{\cot x}$$

$$\frac{dy}{dx} = y\left(\ln \cot x - \frac{x \csc^2 x}{\cot x}\right)$$

$$\boxed{\frac{dy}{dx} = (\cot x)^x\left(\ln \cot x - \frac{x\csc^2 x}{\cot x}\right)}$$

(7) (20 points) Use the definition of the derivative (i.e. set up and evaluate a limit)[1] to show that $\frac{d}{dx}x^4 = 4x^3$.

$$\frac{d}{dx} x^4 = \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$$

using footnote

$$= \lim_{h \to 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$

$$= \lim_{h \to 0} \frac{h(4x^3 + 6x^2h + 4xh^2 + h^3)}{h}$$

$$= \lim_{h \to 0} 4x^3 + 6x^2h + 4xh^2 + h^3$$

$$= 4x^3 + 6x^2 \cdot 0 + 4x \cdot 0^2 + 0^3$$

$$= 4x^3$$

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[1] The following may be very useful:

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4.$$