Exam 2 Math 1190 sec. 51

Summer 2017

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	

INSTRUCTIONS: You have 60 minutes to complete this exam.

There are 8 problems. The point values are listed with the problems.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) (10 points) Evaluate each basic derivative. (No *work* needs to be shown. These you either know or don't know.)

a) $\frac{d}{dx}\ \cos x =$ $-\sin x$

b) $\frac{d}{dx}\ e^{-x} =$ $-e^{-x}$

c) $\frac{d}{dx}\ \sin^{-1} x =$ $\frac{1}{\sqrt{1-x^2}}$

d) $\frac{d}{dx}\ x^5 =$ $5x^4$

e) $\frac{d}{dx}\ \tan x =$ $\sec^2 x$

f) $\frac{d}{dx}\ 3 =$ 0

g) $\frac{d}{dx}\ \tan^{-1} x =$ $\frac{1}{1+x^2}$

h) $\frac{d}{dx}\ \sqrt{x} =$ $\frac{1}{2\sqrt{x}}$

i) $\frac{d}{dx}\ \frac{1}{x} =$ $\frac{-1}{x^2}$

j) $\frac{d}{dx}\ 6^x =$ $6^x \ln 6$

(2) (20 points) Evaluate each derivative using any applicable derivative rules. (It is not necessary to simplify.)

a) $\dfrac{d}{dx}\, x \ln x =$ $\ln x + x \cdot \frac{1}{x} = \ln x + 1$

b) $\dfrac{d}{dx}\, \dfrac{x^2+2x}{x-3} =$ $\dfrac{(2x+2)(x-3) - (x^2+2x)}{(x-3)^2}$

c) $\dfrac{d}{dx}\, \sin(e^x) =$ $\cos(e^x)\, e^x$

d) $\dfrac{d}{dx}\, \ln(3x^2+2) =$ $\dfrac{6x}{3x^2+2}$

e) $\dfrac{d}{dx}\, \dfrac{1}{\sqrt[3]{x^2}} =$ $\frac{-2}{3}\, x^{-5/3}$

$x^{-2/3}$

(3) (8 points) Find $f'(x)$. It is not necessary to simplify. (Hint: Log properties are your friend!)

$$f(x) = \ln\left(\frac{x(x-2)^3}{\sin^2 x}\right) = \ln x + 3\ln(x-2) - 2\ln(\sin x)$$

$$f'(x) = \frac{1}{x} + \frac{3}{x-2} - 2\frac{\cos x}{\sin x}$$

(4) (12 points) Find the first, second, and third derivatives of $f(x) = \dfrac{x}{x+1}$.

$$f'(x) = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}$$ Note $f'(x) = (x+1)^{-2}$

$$f''(x) = -2(x+1)^{-3} = \frac{-2}{(x+1)^3}$$

$$f'''(x) = \frac{6}{(x+1)^4}$$

(5) (a) (7 points) Find $\frac{dy}{dx}$ given the relation $x^3y^2 + 3y = -2$.

$$3x^2y^2 + 2x^3y \frac{dy}{dx} + 3\frac{dy}{dx} = 0$$

$$(2x^3y + 3)\frac{dy}{dx} = -3x^2y^2$$

$$\frac{dy}{dx} = \frac{-3x^2y^2}{2x^3y+3}$$

(b) (3 points) Find the equation of the line tangent to the graph of the given relation at the point $(1, -1)$. Express your answer in the form $y = mx + b$.

when $x=1$ and $y=-1$, $m_{tan} = \frac{dy}{dx}$ @ $(1,-1)$

$$m_{tan} = \frac{-3(1)^2(-1)^2}{2(1)^3(-1)+3} = \frac{-3}{1} = -3$$

$$y-(-1) = -3(x-1)$$

$$y = -3x + 3 - 1$$

$$\boxed{y = -3x + 2}$$

(6) When studying a certain elevator, it was determined that riders experienced the most comfortable ride when the vertical motion is given by $s(t) = 4t + \frac{4}{5}t^2 + \frac{1}{3}t^3$. The position (height) s is in meters and time t is in seconds.

(a) (5 points) Find the velocity $v(t)$ of the elevator (include units in your answer).

$$v(t) = s'(t) = 4 + \frac{8}{5}t + t^2 \quad \frac{m}{sec}$$

(b) (5 points) Find the acceleration of the elevator when $t = 1$ second. Include units in your answer.

$$a(t) = v'(t) = \frac{8}{5} + 2t \quad \frac{m}{sec^2}$$

$$\Rightarrow \quad a(1) = \frac{8}{5} + 2 \ \frac{m}{sec^2} = \frac{18}{5} \ \frac{m}{sec^2}$$

(c) (2 points) The rate of change of acceleration is called *jerk*. Find the jerk J of the elevator.

$J = a'(t) = 2$ The jerk $J = 2 \ \frac{m}{sec^3}$

(7) Suppose f and g are differentiable functions and we know that

$f(-1) = 3,$	$f(0) = 4,$	$f(1) = 2,$	$f'(-1) = 7,$	$f'(0) = 3,$	$f'(1) = -4$
$g(-1) = 2,$	$g(0) = -2,$	$g(2) = 6,$	$g'(-1) = -1,$	$g'(0) = 2,$	$g'(2) = -3$

Find if possible

(a) (2 pts) If $y = f(x) + 3g(x)$, find $y'(-1)$ $\quad y'(-1) = f'(-1) + 3g'(-1) = 7 + 3(-1) = \underline{\underline{4}}$

(b) (2 pts) if $F(x) = f(x)g(x)$, find $F'(0)$ $\quad F'(0) = f'(0)g(0) + f(0)g'(0)$

$= 3 \cdot (-2) + 4(2) = \underline{\underline{2}}$

(c) (3 pts) If $z(x) = g \circ f$, find $z'(1)$

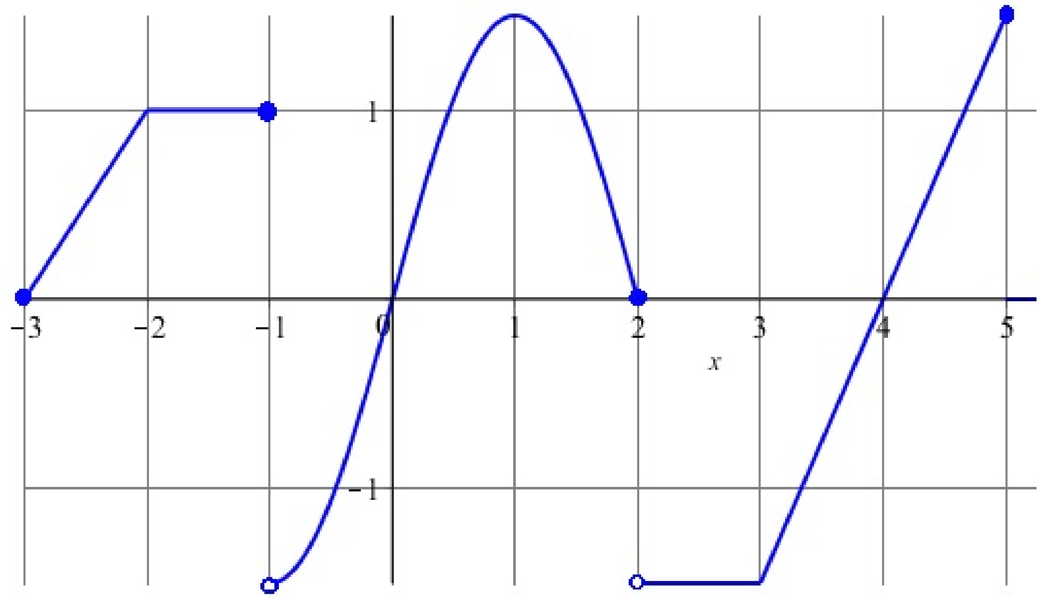
$$z'(1) = g'(f(1)) \cdot f'(1) = g'(2) f'(1) = -3 \cdot (-4) = \underline{\underline{12}}$$

(d) (3 pts) The equation of the line tangent to the graph of g at the point $(2, g(2))$.

$g(2) = 6 \qquad m_{tan} = g'(2) = -3$

$$y - 6 = -3(x-2) \Rightarrow \underline{\underline{y = -3x + 12}}$$

(8) Use the graph of $y = f(x)$ shown in the figure to evaluate or otherwise answer each question.



a) (3 pts) Evaluate (or write DNE) $f'(-2) =$ DNE (corner)

b) (3 pts) Evaluate (or write DNE) $f'(1) =$ 0 (horizontal tangent)

c) (3 pts) Evaluate (or write DNE) $f'(2) =$ DNE (jump)

d) (3 pts) Is $f'(x)$ positive, negative, or zero on the interval $3 < x < 5$?

positive (slope upward)

e) (3 pts) Identify all x-value(s) at which f is continuous but f is not differentiable.

$x=-2$ and $x=3$ (corners)

f) (3 pts) Write the three quantities in order from least to greatest: $f'\left(-\frac{3}{2}\right)$, $f'(0)$, $f'\left(\frac{3}{2}\right)$

0 + −

positive negative

$f'(\frac{3}{2})$	$f'(\frac{-3}{2})$	$f'(0)$
smallest	middle	largest