Exam 2 Math 1190 sec. 52

Fall 2016

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	

INSTRUCTIONS: There are 7 problems. The point values are listed with the problems.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.**

To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) (15 points, 5 each) Find the derivative of each function. Do not leave compound fractions in your answers, otherwise it is not necessary to simplify.

(a) $y = x^3 \sin x$

$y' = 3x^2 \sin x + x^3 \cos x$

(b) $f(x) = \dfrac{\cot x}{x^2 - 1}$

$$f'(x) = \frac{-\csc^2 x \, (x^2-1) - \cot x \, (2x)}{(x^2-1)^2}$$

$$= -\frac{(x^2-1)\csc^2 x - 2x \cot x}{(x^2-1)^2}$$

(c) $y = \sqrt[4]{x^3} = x^{3/4}$

$y' = \frac{3}{4} x^{-1/4} = \dfrac{3}{4\sqrt[4]{x}}$

(2) (15 points) Find $\frac{dy}{dx}$ given the relation $xy^2 = 3x - y^4$.

$$y^2 + x(2y)\frac{dy}{dx} = 3 - 4y^3\frac{dy}{dx}$$

$$2xy\frac{dy}{dx} + 4y^3\frac{dy}{dx} = 3 - y^2$$

$$(2xy + 4y^3)\frac{dy}{dx} = 3 - y^2$$

$$\boxed{\frac{dy}{dx} = \frac{3-y^2}{2xy+4y^3}}$$

(3) (15 points) Find the equation of the line tangent to the graph of $f(x) = 2xe^x$ at the point $(1, 2e)$.

$$f'(x) = 2e^x + 2xe^x \qquad f'(1) = 2e^1 + 2\cdot 1 e^1 = 4e$$

$$y - 2e = 4e(x-1) = 4ex - 4e$$

$$y = 4ex - 4e + 2e$$

$$\boxed{y = 4ex - 2e}$$

(4) (15 points, 5 each) Find the derivative of each function.

(a) $f(x) = \tan^{-1}(x^2)$

$$f'(x) = \frac{1}{1+(x^2)^2}(2x) = \frac{2x}{1+x^4}$$

(b) $f(x) = x\sin^{-1}x$

$$f'(x) = \sin^{-1}x + x\frac{1}{\sqrt{1-x^2}}$$

$$= \sin^{-1}x + \frac{x}{\sqrt{1-x^2}}$$

(c) $g(t) = 4^t + \log_4(t)$

$$g'(t) = 4^t \ln 4 + \frac{1}{t\ln 4}$$

(5) (10 points) Find the x-value of all points on the graph of the function at which the tangent line is horizontal.

$f(x) = (x+1)^3(x-2)^{-4}$

$f'(x) = 3(x+1)^2(x-2)^{-4} - 4(x+1)^3(x-2)^{-5}$

$3(x+1)^2(x-2)^{-4} - 4(x+1)^3(x-2)^{-5} = 0$

$(x+1)^2(x-2)^{-5}\left[3(x-2) - 4(x+1)\right] = 0$

$(x+1)^2(x-2)^{-5}(-x-10) = 0 \Rightarrow x = -1$ or $x = -10$

There are horizontal tangents @ $x = -1$ and @ $x = -10$.

(6) (10 points) Use logarithmic differentiation to find $\frac{dy}{dx}$. Assume y is positive. (Your final result should be stated in terms of x, but need not be fully simplified.)

$y = (\cos x)^x$

$\ln y = \ln(\cos x)^x = x \ln \cos x$

$\frac{1}{y}\frac{dy}{dx} = \ln \cos x + x\frac{-\sin x}{\cos x}$

$\frac{dy}{dx} = y(\ln \cos x - x\tan x)$

$$\boxed{\frac{dy}{dx} = (\cos x)^x(\ln \cos x - x\tan x)}$$

(7) (20 points) Use the definition of the derivative (i.e. set up and evaluate a limit)[1] to show that $\frac{d}{dx}x^5 = 5x^4$.

$$\frac{d}{dx} x^5 = \lim_{h\to 0} \frac{(x+h)^5 - x^5}{h}$$

$$= \lim_{h\to 0} \frac{x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5 - x^5}{h}$$

$$= \lim_{h\to 0} \frac{5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5}{h}$$

$$= \lim_{h\to 0} \frac{h(5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4)}{h}$$

$$= \lim_{h\to 0} 5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4$$

$$= 5x^4 + 10x^3 \cdot 0 + 10x^2 \cdot 0^2 + 5x \cdot 0^3 + 0^4$$

$$= 5x^4$$

[1] The following may be very useful:

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5.$$