Exam 2 Math 1190 sec. 62

Spring 2017

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	

INSTRUCTIONS: You have 50 minutes to complete this exam.

There are 8 problems. The point values are listed with the problems.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) **Use the definition of the derivative** (i.e. set up and evaluate a limit) to find $f'(1)$ where $f(x) = 2x^2 + x$. (Use of the power or other *derivative rules* will not be considered for credit.)

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{2(1+h)^2 + (1+h) - 3}{h}$$

$$= \lim_{h\to 0} \frac{2(1+2h+h^2) + 1 + h - 3}{h}$$

$$= \lim_{h\to 0} \frac{2+4h+2h^2 + 1+h-3}{h}$$

$$= \lim_{h\to 0} \frac{3+5h+2h^2-3}{h} = \lim_{h\to 0} \frac{h(5+2h)}{h}$$

$$= \lim_{h\to 0} (5+2h) = 5$$

(2) Find the equation of the line tangent to the graph of $f(x) = \tan^{-1}(x)$ at the point $\left(1, \frac{\pi}{4}\right)$. Express your answer in the form $y = mx + b$.

$$m = f'(1) \Rightarrow f'(x) = \frac{1}{1+x^2}, \quad m = \frac{1}{1+1^2} = \frac{1}{2}$$

$$y - \frac{\pi}{4} = \frac{1}{2}(x-1)$$

$$\boxed{y = \frac{1}{2}x - \frac{1}{2} + \frac{\pi}{4}}$$

(3) Use derivative rules to find $\frac{dy}{dx}$. (Simplification is not required.)

(a) $y = x^7 + 3x^3 - 4$

$y' = 7x^6 + 9x^2$

(b) $y = \dfrac{\tan x}{x^3 + 1}$

$$y' = \frac{\sec^2 x\,(x^3+1) - \tan x\,(3x^2)}{(x^3+1)^2}$$

$$= \frac{(x^3+1)\sec^2 x - 3x^2 \tan x}{(x^3+1)^2}$$

(c) $y = e^x \sec x$

$y' = e^x \sec x + e^x \sec x \tan x$

(4) Find $\frac{dy}{dx}$ given the relation $x^3 y^4 = 2x + e^y$

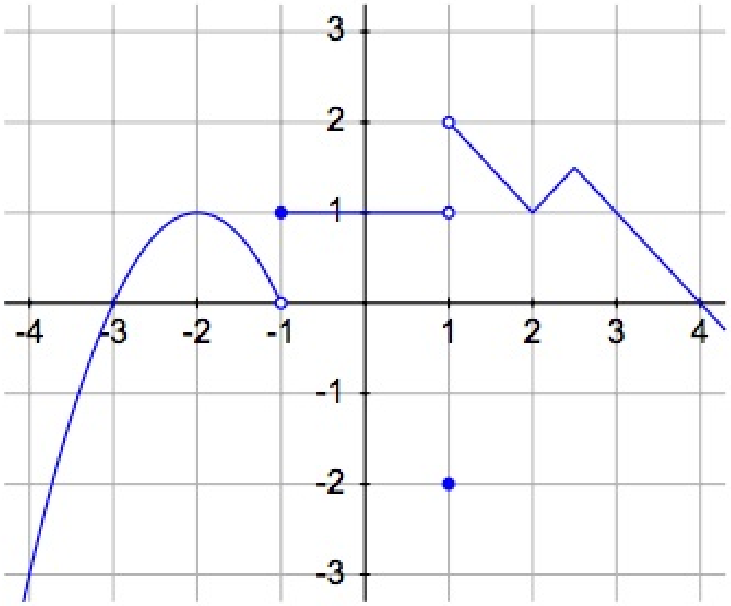
$$3x^2 y^4 + x^3 (4y^3) \frac{dy}{dx} = 2 + e^y \frac{dy}{dx}$$

$$4x^3 y^3 \frac{dy}{dx} - e^y \frac{dy}{dx} = 2 - 3x^2 y^4$$

$$(4x^3 y^3 - e^y) \frac{dy}{dx} = 2 - 3x^2 y^4$$

$$\frac{dy}{dx} = \frac{2 - 3x^2 y^4}{4x^3 y^3 - e^y}$$

(5) Use the graph of $y = f(x)$ shown to evaluate or answer the following questions.



a Evaluate if possible (if not, write DNE) $f'(-2) =$ 0

b Evaluate if possible (if not, write DNE) $f'(2) =$ DNE

c Evaluate if possible (if not, write DNE) $f'(0) =$ 0

d Is $f'(-3)$ positive, negative, or zero? positive

e Which is greater $f(3)$ or $f'(3)$? (Justify your claim.)

$f(3) = 1$, $f'(3)$ is negative

so $f(3)$ is greater than $f'(3)$

(6) Suppose f and g are differentiable functions and we know that

$$f(-1) = 1, \quad f(0) = 2, \quad f(2) = 4, \quad f'(-1) = -2, \quad f'(0) = 4, \quad f'(2) = 3$$

$$g(0) = 1, \quad g(1) = -2, \quad g(4) = -1, \quad g'(0) = 3, \quad g'(1) = -2, \quad g'(4) = \frac{1}{2}$$

a If $y = f(x)g(x)$, find $y'(0)$.

$$y'(0) = f'(0)\,g(0) + f(0)\,g'(0)$$
$$= 4 \cdot 1 + 2 \cdot 3 = 10$$

b If $F(x) = \frac{f(x)}{g(x)}$, find $F'(0)$.

$$F'(0) = \frac{f'(0)\,g(0) - f(0)\,g'(0)}{(g(0))^2} = \frac{4 \cdot 1 - 2 \cdot 3}{1^2} = -2$$

c If $h = f \circ g$, find $h'(4)$.

$$h'(4) = f'(g(4)) \cdot g'(4)$$
$$= f'(-1)\,g'(4) = -2 \cdot \tfrac{1}{2} = -1$$

d If $s = g \circ f$, find $s'(-1)$.

$$s'(-1) = g'(f(-1)) \cdot f'(-1)$$
$$= g'(1)\,f'(-1) = -2 \cdot (-2) = 4$$

(7) Use derivative rules to find $\frac{dy}{dx}$. (Simplification is not necessary.)

(a) $y = \sec(e^x)$

$y' = \sec(e^x)\tan(e^x)\cdot e^x$

$= e^x \sec(e^x)\tan(e^x)$

(b) $y = \dfrac{\sqrt{x}+1}{x} = x^{-1/2} + x^{-1}$

$y' = \frac{-1}{2}x^{-3/2} - x^{-2} = \frac{-1}{2x^{3/2}} - \frac{1}{x^2}$

(c) $y = \sin^{-1}(3^x)$

$y' = \dfrac{1}{\sqrt{1-(3^x)^2}} \cdot 3^x \ln 3 = \dfrac{3^x \ln 3}{\sqrt{1-3^{2x}}}$

(8) (10 points) A squirrel runs up and down a telephone pole. His position after t seconds is $s(t) = 8\cos(t) + 6\sin(t)$ feet.

(a) Determine his velocity $v(t)$. What are the units?

$v(t) = s'(t) = -8\sin t + 6\cos t \quad \frac{ft}{sec}$

(b) Determine his acceleration after $\frac{\pi}{2}$ seconds. Include units in your answer.

$a(t) = v'(t) = -8\cos t - 6\sin t \quad \frac{ft}{sec^2}$

$a(\frac{\pi}{2}) = -8\cos\frac{\pi}{2} - 6\sin\frac{\pi}{2} = -6 \quad \frac{ft}{sec^2}$