Exam 2 Math 1190 sec. 63

Spring 2017

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	

INSTRUCTIONS: You have 50 minutes to complete this exam.

There are 8 problems. The point values are listed with the problems.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) (12 points) **Use the definition of the derivative** (i.e. set up and evaluate a limit) to find $f'(1)$ where $f(x) = 2x - x^2$. (Use of the power or other *derivative rules* will **NOT** be considered for credit.)

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{2(1+h) - (1+h)^2 - 1}{h}$$

$$= \lim_{h\to 0} \frac{2+2h - (1+2h+h^2) - 1}{h}$$

$$= \lim_{h\to 0} \frac{2+2h-1-2h-h^2-1}{h}$$

$$= \lim_{h\to 0} \frac{-h^2}{h} = \lim_{h\to 0} -h = 0$$

(2) (15 points) Find the equation of the line tangent to the graph of $f(x) = \tan^{-1}(x)$ at the point $\left(-1, -\frac{\pi}{4}\right)$. Express your answer in the form $y = mx + b$.

$$m = f'(1) \qquad f'(x) = \frac{1}{1+x^2}, \qquad m = \frac{1}{1+(-1)^2} = \frac{1}{2}$$

$$y - \left(\frac{-\pi}{4}\right) = \frac{1}{2}(x - (-1))$$

$$\boxed{y = \frac{1}{2}x + \frac{1}{2} - \frac{\pi}{4}}$$

(3) (15 points) Use derivative rules to find $\frac{dy}{dx}$. (Simplification is not required.)

(a) $y = 4x^5 + 2x^4 - 6x$

$y' = 20x^4 + 8x^3 - 6$

(b) $y = \dfrac{\sin x}{x^4 + 4}$

$$y' = \frac{\cos x\,(x^4+4) - \sin x\,(4x^3)}{(x^4+4)^2}$$

$$= \frac{(x^4+4)\cos x - 4x^3 \sin x}{(x^4+4)^2}$$

(c) $y = e^x \cot x$

$y' = e^x \cot x - e^x \csc^2 x$

(4) (15 points) Find $\frac{dy}{dx}$ given the relation $xe^y = x^2y^2 + 1$

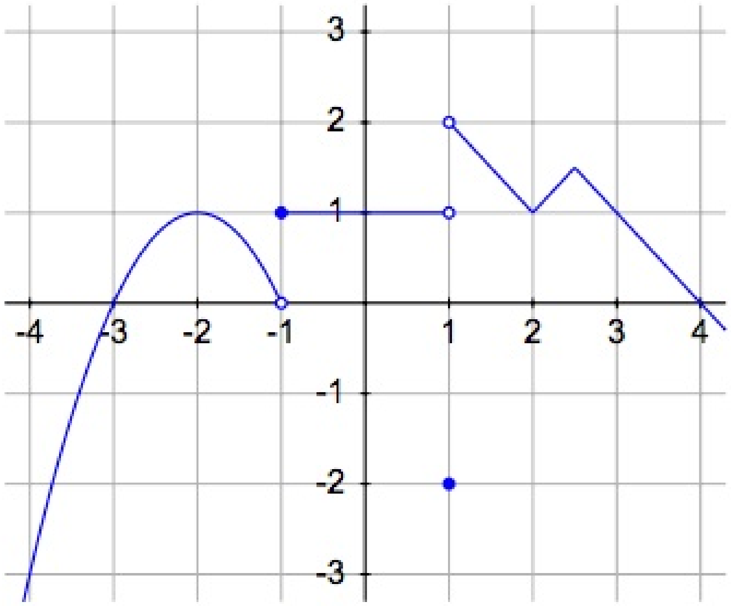
$$1e^y + xe^y \cdot \frac{dy}{dx} = 2xy^2 + x^2\left(2y\frac{dy}{dx}\right)$$

$$xe^y \frac{dy}{dx} - 2x^2y\frac{dy}{dx} = 2xy^2 - e^y$$

$$(xe^y - 2x^2y)\frac{dy}{dx} = 2xy^2 - e^y$$

$$\boxed{\frac{dy}{dx} = \frac{2xy^2 - e^y}{xe^y - 2x^2y}}$$

(5) (10 points) Use the graph of $y = f(x)$ shown to evaluate or answer the following questions.



a Evaluate if possible (if not, write DNE) $f'(-1) =$ DNE (Discontinuity)

b Evaluate if possible (if not, write DNE) $f'(-2) =$ 0

c Evaluate if possible (if not, write DNE) $f'(0) =$ 0

d Is $f'(3)$ positive, negative, or zero? Negative (sloped downward)

e Which is greater $f(-3)$ or $f'(-3)$? (Justify your claim.)

f(-3) = 0 and f'(-3) is positive

so f'(-3) is greater than f(-3)

(6) (8 points) Suppose f and g are differentiable functions and we know that

$$f(-1) = 2, \quad f(0) = -3, \quad f(2) = 5, \quad f'(-1) = 1, \quad f'(0) = 4, \quad f'(2) = 3$$

$$g(0) = -1, \quad g(2) = 3, \quad g(5) = 2, \quad g'(0) = \frac{1}{3}, \quad g'(2) = 2, \quad g'(5) = 4$$

a If $y = f(x)g(x)$, find $y'(0)$.

$$y'(0) = f'(0)\,g(0) + f(0)\,g'(0)$$
$$= 4 \cdot (-1) + (-3) \cdot \tfrac{1}{3} = -4 - 1 = -5$$

b If $F(x) = \frac{f(x)}{g(x)}$, find $F'(0)$.

$$F'(0) = \frac{f'(0)\,g(0) - f(0)\,g'(0)}{(g(0))^2} = \frac{4 \cdot (-1) - (-3)(\frac{1}{3})}{(-1)^2}$$
$$= -4 + 1 = -3$$

c If $h = f \circ g$, find $h'(5)$.

$$h'(5) = f'(g(5))\,g'(5)$$
$$= f'(2)\,g'(5) = 3 \cdot 4 = 12$$

d If $s = g \circ f$, find $s'(-1)$.

$$s'(-1) = g'(f(-1)) \cdot f'(-1)$$
$$= g'(2)\,f'(-1) = 2 \cdot 1 = 2$$

(7) (15 points) Use derivative rules to find $\frac{dy}{dx}$. (Simplification is not necessary.)

(a) $y = e^{\tan x}$

$y' = e^{\tan x} \cdot \sec^2 x = \sec^2 x \; e^{\tan x}$

(b) $y = \dfrac{x^2 - \sqrt[4]{x}}{x} = \dfrac{x^2}{x} - \dfrac{x^{1/4}}{x} = x - x^{-3/4}$

$y' = 1 + \frac{3}{4} x^{-7/4} = 1 + \dfrac{3}{4x^{7/4}}$

(c) $y = 4^x \sin^{-1}(x)$

$y' = 4^x(\ln 4) \sin^{-1} x + \dfrac{4^x}{\sqrt{1-x^2}}$

(8) (10 points) A squirrel runs up and down a telephone pole. His position after t seconds is $s(t) = 5 - 4\cos(t)$ feet.

(a) Determine his velocity $v(t)$. What are the units?

$v(t) = s'(t) = 4\sin(t)$ ft/sec

(b) Determine his acceleration after π seconds. Include units in your answer.

$a(t) = v'(t) = 4\cos t$ ft/sec^2

$a(\pi) = 4\cos(\pi) = -4$ ft/sec^2