Exam II Math 3260 sec. 55

Spring 2018

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	

INSTRUCTIONS: There are 7 problems; the point values are listed with the problems. You may use a calculator with matrix capabilities. **No wifi enabled device can be used as a calculator.** There are no notes, or books allowed. **Illicit use of a smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, you must clearly justify your answer.

(1) (30 points, 5 each) Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be defined by $T(x_1, x_2) = (x_1 - x_2, 2x_2 + x_1, 0)$

$\mathbb{R}^2$ $\mathbb{R}^3$

(a) Identify the domain and codomain of T.

domain $\mathbb{R}^2$

codomain $\mathbb{R}^3$

(b) Find the standard matrix A for the linear transformation T.

$T(1,0) = (1,1,0)$

$T(0,1) = (-1,2,0)$

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 2 \\ 0 & 0 \end{bmatrix}$$

(c) Is $(1, 1)$ in the domain of T? If so, find its image.

Yes, $T(1,1) = (0,3,0)$

(d) Is $(1, 1)$ in the range of T? If so, find $\mathbf{x}$ in the domain such that $T(\mathbf{x}) = (1, 1)$.

No, the range is in $\mathbb{R}^3$ not $\mathbb{R}^2$.

(e) Is $(1, 1, 0)$ in the domain of T? If so, find its image.

No, $(1,1,0)$ is in $\mathbb{R}^3$ but the domain is $\mathbb{R}^2$

(f) Is $(1, 1, 0)$ in the range of T? If so, find $\mathbf{x}$ in the domain such that $T(\mathbf{x}) = (1, 1, 0)$.

$T(1,0) = (1,1,0)$ so Yes, $(1,1,0)$ is in the range and $(1,0)$ is one such $\vec{x}$.

(2) (10 points) Find all values of x such that $\det(A) = 0$ where $A = \begin{bmatrix} x+1 & -3 & 3 \\ 0 & x-3 & 5 \\ 0 & 1 & x+1 \end{bmatrix}$.

$\det(A) = (x+1) \begin{vmatrix} x-3 & 5 \\ 1 & x+1 \end{vmatrix} = (x+1)((x-3)(x+1) - 5)$

$= (x+1)(x^2 - 2x - 8)$

$= (x+1)(x-4)(x+2)$

$\det(A) = 0$ if $x = -1, 4,$ or -2

(3) (a) (5 points) In your own words, give a definition of Span.

Span of a set of vectors is the set of all linear combinations of those vectors.

(b) (5 points) Suppose $\mathbf{u}$ is a nonzero vector in $\mathbb{R}^n$. What if anything is the difference between the set $\{\mathbf{u}\}$ and Span$\{\mathbf{u}\}$?

$\{\vec{u}\}$ is a set containing on vector.

Span $\{\vec{u}\}$ contains infinitely many vectors of the form $c\vec{u}$ where c is any scaler.

(4) (25 points, 5 each) Use the given matrices to evaluate the desired expression. If the expression is undefined give a reason (more detailed than "because a calculator says so.").

$$A = \begin{bmatrix} 1 & 1 \\ 7 & -2 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 0 & 10 \\ 3 & 3 & 1 \\ 0 & 0 & 6 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 4 \\ -2 & 6 \end{bmatrix}$$

(a) BC

3x3 2x3 Undefined # columns B ≠ # rows C

(b) $D + 3A^T$

$$\begin{bmatrix} 0 & 4 \\ -2 & 6 \end{bmatrix} + 3\begin{bmatrix} 1 & 7 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 25 \\ 1 & 0 \end{bmatrix}$$

(c) $\det(C)$ Undefined C is not a square matrix

(d) CB $= \begin{bmatrix} 4 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 0 & 10 \\ 3 & 3 & 1 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} -9 & 3 & 41 \\ -6 & -6 & 4 \end{bmatrix}$

(e) $\det(B)$ $= 6\begin{vmatrix} -3 & 0 \\ 3 & 3 \end{vmatrix} = 6(-9) = -54$

(5) (10 points) Let $A = \begin{bmatrix} 2 & 4 & 0 \\ 0 & -1 & 1 \end{bmatrix}$. Find a spanning set for $\mathrm{Nul}(A)$.

$$\mathrm{rref}(A) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

$x_1 = -2x_3$
$x_2 = x_3$
x_3 - free

If $\vec{x}$ is in Nul A

$$\vec{x} = x_3 \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

So $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a spanning set.

i.e. $\mathrm{Nul}(A) = \mathrm{Span} \left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$.

(6) (10 points) Suppose we wish to solve the given system using Crammer's rule. Determine the values of the parameter s for which the given system has exactly one solution. Then for those s, find the solution using Crammer's rule.

$$\begin{aligned} sX + 4Y &= 2 \\ 9X + sY &= 1 \end{aligned}$$

$A = \begin{bmatrix} s & 4 \\ 9 & s \end{bmatrix}$

$$\det\left(\begin{bmatrix} s & 4 \\ 9 & s \end{bmatrix} \right) = s^2 - 36 = (s-6)(s+6)$$

$\det(A) \neq 0$ if $s \neq \pm 6$

$$\det(A_1) = \left| \begin{bmatrix} 2 & 4 \\ 1 & s \end{bmatrix} \right| = 2s - 4 \qquad \det(A_2) = \left| \begin{bmatrix} s & 2 \\ 9 & 1 \end{bmatrix} \right| = s - 18$$

For $s \neq \pm 6$

$$X = \frac{2s-4}{s^2-36}, \quad Y = \frac{s-18}{s^2-36}$$

(7) (5 points) Let $T : \mathbb{R}^2 \longrightarrow \mathbb{P}_1$ be the linear transformation defined by

$$T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = a + 2at$$

(a) Evaluate $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$. $= 1+2t$

(b) Evaluate $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$. $= 1+2t$

(c) Evaluate $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$. $= 0+0t = 0$

(d) Is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ in the kernel of T? No, $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) \neq 0+0t$

(e) Is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ in the kernel of T? Yes since $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = 0+0t$