Exam II Math 3260 sec. 56

Spring 2018

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	

INSTRUCTIONS: There are 7 problems; the point values are listed with the problems. You may use a calculator with matrix capabilities. **No wifi enabled device can be used as a calculator.** There are no notes, or books allowed. **Illicit use of a smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, you must clearly justify your answer.

(1) (30 points, 5 each) Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be defined by $T(x_1, x_2, x_3) = (3x_2, x_3 - 2x_1)$

$\mathbb{R}^3$ $\mathbb{R}^2$

(a) Identify the domain and codomain of T.

domain $\mathbb{R}^3$

codomain $\mathbb{R}^2$

(b) Find the standard matrix A for the linear transformation T.

$T(1,0,0) = (0,-2)$

$T(0,1,0) = (3,0)$

$T(0,0,1) = (0,1)$

$$A = \begin{bmatrix} 0 & 3 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

(c) Is $(1, 1)$ in the domain of T? If so, find its image.

No, $(1,1)$ is in $\mathbb{R}^2$ but the domain is $\mathbb{R}^3$

(d) Is $(1, 1)$ in the range of T? If so, find $\mathbf{x}$ in the domain such that $T(\mathbf{x}) = (1, 1)$.

Yes, $T(x_1, x_2, x_3) = (1,1)$ if $x_2 = \frac{1}{3}$, $x_3 = 1$, $x_1 = 0$

$\vec{x} = (0, \frac{1}{3}, 1)$ is one answer.

(e) Is $(1, 1, 0)$ in the domain of T? If so, find its image.

Yes, $T(1,1,0) = (3,-2)$

(f) Is $(1, 1, 0)$ in the range of T? If so, find $\mathbf{x}$ in the domain such that $T(\mathbf{x}) = (1, 1, 0)$.

No, the range is in $\mathbb{R}^2$ but $(1,1,0)$ is in $\mathbb{R}^3$

(2) (10 points) Find all values of x such that $\det(A) = 0$ where $A = \begin{bmatrix} x-1 & 2 & -2 \\ 0 & x+1 & 1 \\ 0 & 5 & x-3 \end{bmatrix}$.

$$\det(A) = (x-1)\begin{vmatrix} x+1 & 1 \\ 5 & x-3 \end{vmatrix}$$

$$= (x-1)\left((x+1)(x-3) - 5\right)$$

$$= (x-1)\left(x^2 - 2x - 8\right)$$

$$= (x-1)(x-4)(x+2)$$

$\det(A) = 0$ if $x = 1$, $x = 4$ or $x = -2$

(3) (a) (5 points) In your own words, give a definition of Span.

Span of a set of vectors is the set of all linear combinations of those vectors.

(b) (5 points) Suppose $\mathbf{u}$ is a nonzero vector in $\mathbb{R}^n$. What if anything is the difference between the set $\{\mathbf{u}\}$ and $\text{Span}\{\mathbf{u}\}$?

$\{\vec{u}\}$ is a set containing one vector

$\text{Span}\{\vec{u}\}$ is a set of infinitely many vectors all of the form $c\vec{u}$ for scalars c.

(4) (25 points, 5 each) Use the given matrices to evaluate the desired expression. If the expression is undefined give a reason (more detailed than "because a calculator says so.").

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ 1 & 0 & 6 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 5 & 1 \\ -2 & 1 \end{bmatrix}$$

(a) BC

3×3 2×3 Undefined # Columns B $\neq$ # rows C

(b) $D + 3A^T$ $= \begin{bmatrix} 5 & 1 \\ -2 & 1 \end{bmatrix} + 3\begin{bmatrix} 2 & 1 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 11 & 4 \\ -11 & 1 \end{bmatrix}$

(c) $\det(C)$ Undefined, C is not square

(d) CB $= \begin{bmatrix} 4 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ 1 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -2 & -3 \\ 2 & -1 & 6 \end{bmatrix}$

(e) $\det(B)$ $= 1\begin{vmatrix} 2 & 3 \\ 0 & 6 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 1 & 6 \end{vmatrix}$

$12 + 9 = 21$

(5) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{bmatrix}$. Find a spanning set for Nul(A).

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 4 \end{bmatrix} \qquad \begin{aligned} x_1 &= 5x_3 \\ x_2 &= -4x_3 \\ x_3 &- \text{free} \end{aligned}$$

For $\vec{x}$ in Nul A

$$\vec{x} = x_3 \begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix}$$

So $\left\{ \begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix} \right\}$ is a spanning set,

that is

$$\text{Nul}A = \text{Span}\left\{ \begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix} \right\}.$$

(6) (10 points) Suppose we wish to solve the given system using Crammer's rule. Determine the values of the parameter s for which the given system has exactly one solution. Then for those s, find the solution using Crammer's rule.

$$\begin{array}{rcrcl} sX & - & 2Y & = & 3 \\ -32X & + & sY & = & 1 \end{array} \qquad A = \begin{bmatrix} s & -2 \\ -32 & s \end{bmatrix}$$

$\det(A) = s^2 - 64 \qquad \det(A) \neq 0$ if $s \neq \pm 8$

$\det(A_1) = \begin{vmatrix} 3 & -2 \\ 1 & s \end{vmatrix} = 3s + 2$, $\quad \det(A_2) = \begin{vmatrix} s & 3 \\ -32 & 1 \end{vmatrix} = s + 96$

For $s \neq \pm 8$

$$X = \frac{3s+2}{s^2-64}, \quad Y = \frac{s+96}{s^2-64}$$

(7) (5 points) Let $T : \mathbb{R}^2 \longrightarrow \mathbb{P}_1$ be the linear transformation defined by

$$T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = b - 3bt$$

(a) Evaluate $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$. $= 1 - 3t$

(b) Evaluate $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$. $= 0 + 0t = 0$

(c) Evaluate $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$. $= 1 - 3t$

(d) Is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ in the kernel of T? Yes since $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = 0$

(e) Is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ in the kernel of T? No since $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) \neq 0$