Exam 3 Math 1112 sec. 54 Spring 2019

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: _____

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

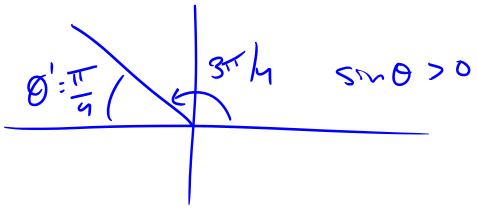
INSTRUCTIONS: There are 10 problems; point values are listed with the problems. There are no notes or books, allowed and **no calculator is allowed on the first 8 problems of this exam. Illicit use of a calculator, smart phone, tablet, device that runs apps, or notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** Upon completing the first 8 problems, turn in this portion of the test to receive the last 2 problems. A non-CAS calculator may be used for the last 2 problems. To receive full credit, answers must be clear, complete, and written using proper notation.

1. (15 points) Fill in the missing values in the table of trigonometric values for select angles. You may wish to do this from memory or by making use of convenient right triangles. This table may serve as reference as you complete other parts of this exam.

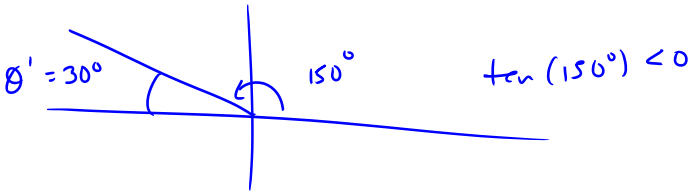
$\theta°$	$0°$	$30°$	$45°$	$60°$	$90°$
θ radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

2. (12 points, 4 each) Evaluate each trigonometric expression exactly.

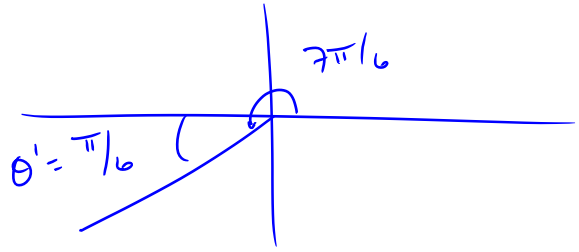
(a) $\sin\left(\frac{3\pi}{4}\right)$ = $+\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$



(b) $\tan(150°)$ = $-\tan(30°) = \frac{-1}{\sqrt{3}}$



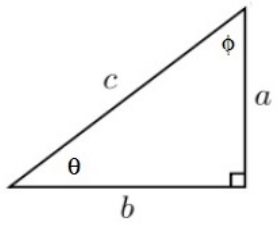
(c) $\sec\left(\frac{7\pi}{6}\right)$ = $\frac{1}{\cos\left(\frac{7\pi}{6}\right)} = \frac{-1}{\cos(\pi/6)} = \frac{-1}{\sqrt{3}/2} = \frac{-2}{\sqrt{3}}$



3. (3 points) Explain what it means for an angle to be in **standard position**.

An angle in standard position in the xy-plane has vertex at the origin and initial ray along the +x-axis.

4. (20 points) Refer to the triangle shown with angles and sides labeled.



(a) (6 pts) Evaluate the sine, cosine, and tangent of θ. (Your answers will contain the variables, a, b and c.)

$\sin\theta = \frac{a}{c}$ $\quad \tan\theta = \frac{a}{b}$

$\cos\theta = \frac{b}{c}$

(b) (6 pts) Evaluate the sine, cosine, and tangent of ϕ. (Your answers will contain the variables, a, b and c.)

$\sin\phi = \frac{b}{c}$

$\cos\phi = \frac{a}{c}$ $\quad \tan\phi = \frac{b}{a}$

(c) (4 pts) Set up and simplify as much as possible. (Your final answer will be a number. Answers without supporting work will not be considered.)

$(\sin\theta)(\tan\phi) - \sin\phi$

$\left(\frac{a}{c}\right)\left(\frac{b}{a}\right) - \left(\frac{b}{c}\right) = \frac{b}{c} - \frac{b}{c} = 0$

(d) (4 pts) Set up and simplify as much as possible. (Your final answer will be a number. Answers without supporting work will not be considered.)

$(\sin\theta)^2 + (\sin\phi)^2$

$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2+b^2}{c^2} = \frac{c^2}{c^2} = 1$ since $a^2+b^2=c^2$

5. (8 points, 4 each) In standard position, the terminal side of the angle θ passes through the point $(2, y)$ in the fourth quadrant.

(a) The distance from the origin to the point $(2, y)$ is 5. Find the value of y.

$\sqrt{2^2+y^2} = 5^2 \Rightarrow y^2 = 25-4 = 21$ as $y<0$, $y = -\sqrt{21}$

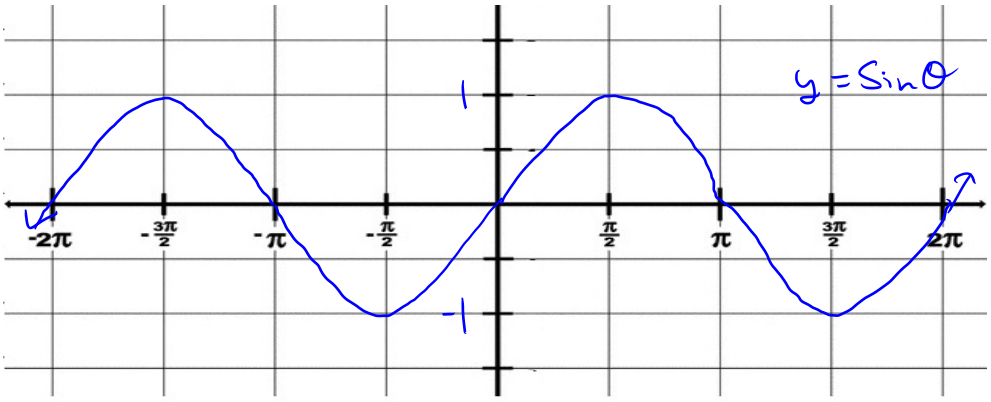
(b) Determine $\sin\theta$, $\cos\theta$, and $\tan\theta$.

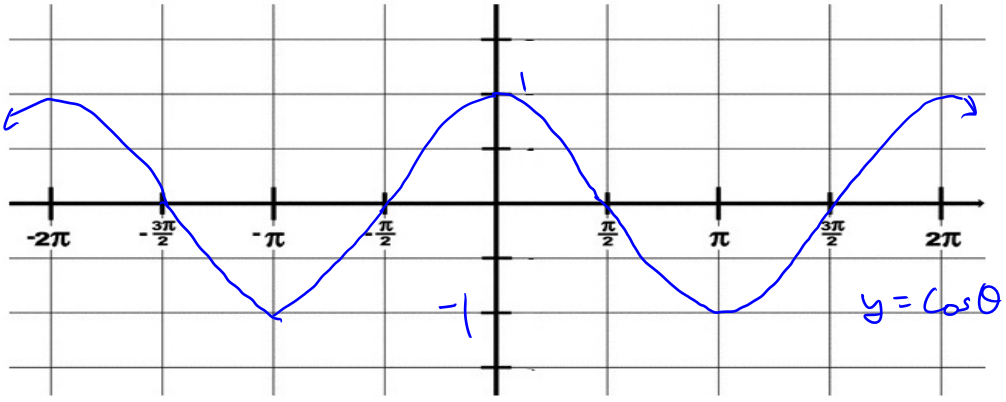
$\sin\theta = \frac{y}{r} = \frac{-\sqrt{21}}{5}$ and $\tan\theta = \frac{y}{x} = \frac{-\sqrt{21}}{2}$

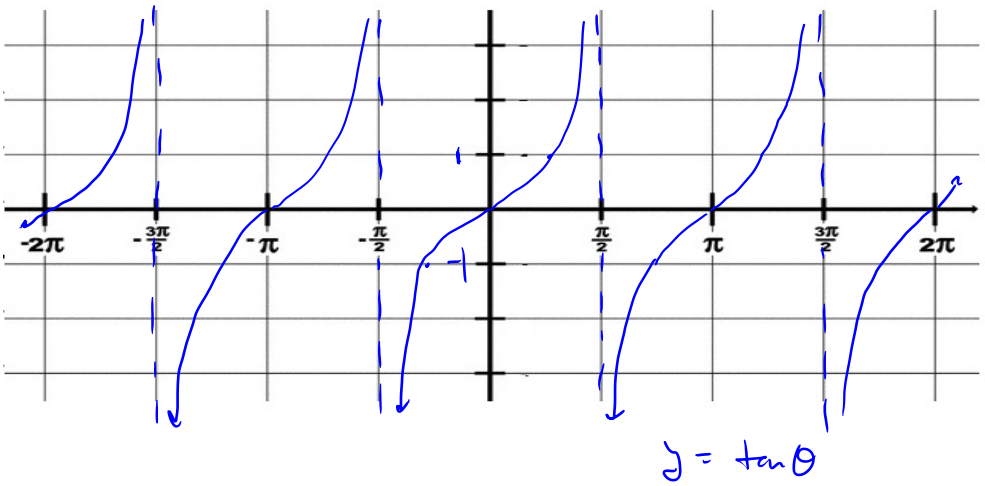
$\cos\theta = \frac{x}{r} = \frac{2}{5}$

6. (12 points, 4 each) Plot at least two full periods of each function on the grids provided. Label the y-axis however you see fit, and indicate on each graph which function you are plotting. Indicate any asymptotes with dashed lines.

(a) $y = \sin x$ (b) $y = \cos x$ (c) $y = \tan x$







7. (10 points, 5 each) Determine the properties of each trigonometric function.

(a) $f(x) = -3\cos(\pi x - \pi) + 2$

i The amplitude $A =$ $|-3| = 3$

ii The period $T =$ $\frac{2\pi}{\pi} = 2$

iii The phase shift is $\frac{\pi}{\pi} = 1$ to the left/right/up/down (circle one)

iv The vertical shift is 2 to the left/right/up/down (circle one)

(b) $f(x) = 4 + 7\sin\left(\frac{x}{2} + \frac{\pi}{4}\right)$

i The amplitude $A =$ $|7| = 7$

ii The period $T =$ $\frac{2\pi}{1/2} = 4\pi$

iii The phase shift is $\frac{\pi/4}{1/2} = \frac{\pi}{2}$ to the left/right/up/down (circle one)

iv The vertical shift is 4 to the left/right/up/down (circle one)

8. (10 points, 2 each) Let $\theta = \sin^{-1}\left(-\frac{1}{4}\right)$. Answer the following questions about θ.

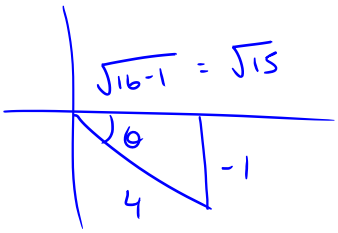
(a) In standard position, in which quadrant would the terminal side of θ lie? 4

(b) Which of the following inequalities involving θ is true? (circle one)

(a) $0 < \theta < \frac{\pi}{2}$, (b) $\frac{3\pi}{2} < \theta < 2\pi$, (c) $-\frac{\pi}{2} < \theta < 0$

(c) What is the value of $\sin\theta$?

$\sin\theta = \frac{-1}{4}$



(d) What is the value of $\cos\theta$?

$\cos\theta = \frac{\sqrt{15}}{4}$

(e) What is the value of $\tan\theta$?

$\tan\theta = \frac{-1}{\sqrt{15}}$

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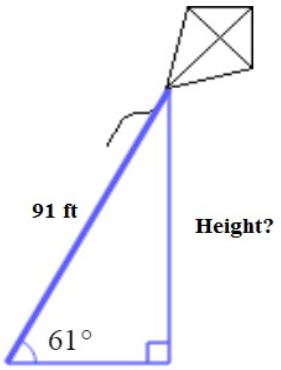
You may use a non-CAS calculator to answer this question.

9. (5 points) A kite is flying with 91 feet of string attached, and the string is taut. If the angle of elevation of the kite is $61°$ as shown in the figure, determine the vertical height of the kite to the nearest tenth of a foot. (You may assume that the string is tethered at ground level.)

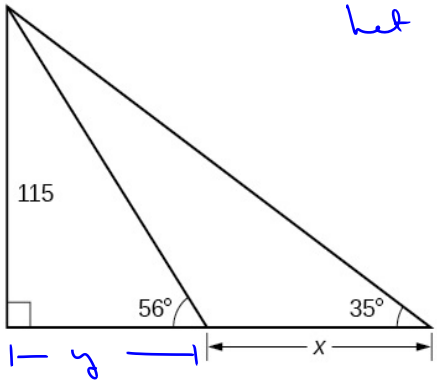


Let h be the height. Then

$$\frac{h}{91\text{ ft}} = \sin 61°$$

$$h = 91 \sin 61° \text{ ft} \approx 79.6 \text{ ft}$$

10. (5 points) Two observers stand x feet apart viewing a building that is 115 feet tall. The close oberver notes an angle of elevation of $56°$, and the farther observer notes an angle of elevation of $35°$ to the top of the building (see the figure.) Determine the distance x to the nearest foot.



Let y be as shown. Then

$$\frac{115}{y} = \tan 56° \Rightarrow y = \frac{115}{\tan 56°}$$

and

$$\frac{115}{x+y} = \tan 35° \Rightarrow x+y = \frac{115}{\tan 35°}$$

Then $$x = \frac{115}{\tan 35°} - y = \frac{115}{\tan 35°} - \frac{115}{\tan 56°}$$

$$\approx 87 \text{ ft}$$