Exam 3 Math 1113 sec. 51 Fall 2018

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

INSTRUCTIONS: There are 10 problems worth 10 points each. **No calculator use is allowed on the first 9 problems of this exam.** Once you have completed the 9 problems, turn this in and receive the last problem. There are no notes, or books allowed. **Illicit use of a smart phone, tablet, device that runs apps, or notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, and written using proper notation.

1. For the given function f, construct and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$.

$f(x) = x^2 - 4x + 10$

$$f(x+h) = (x+h)^2 - 4(x+h) + 10$$
$$= x^2 + 2xh + h^2 - 4x - 4h + 10$$

$$\frac{f(x+h)-f(x)}{h} = \frac{x^2+2xh+h^2-4x-4h+10-(x^2-4x+10)}{h}$$

$$= \frac{2xh+h^2-4h}{h}$$

$$= \frac{h(2x+h-4)}{h}$$

$$= 2x+h-4$$

2. Let $g(x) = \begin{cases} 1-x, & -1 \le x < 1 \\ 2, & x = 1 \\ 2-x, & 1 < x \le 3 \end{cases}$.

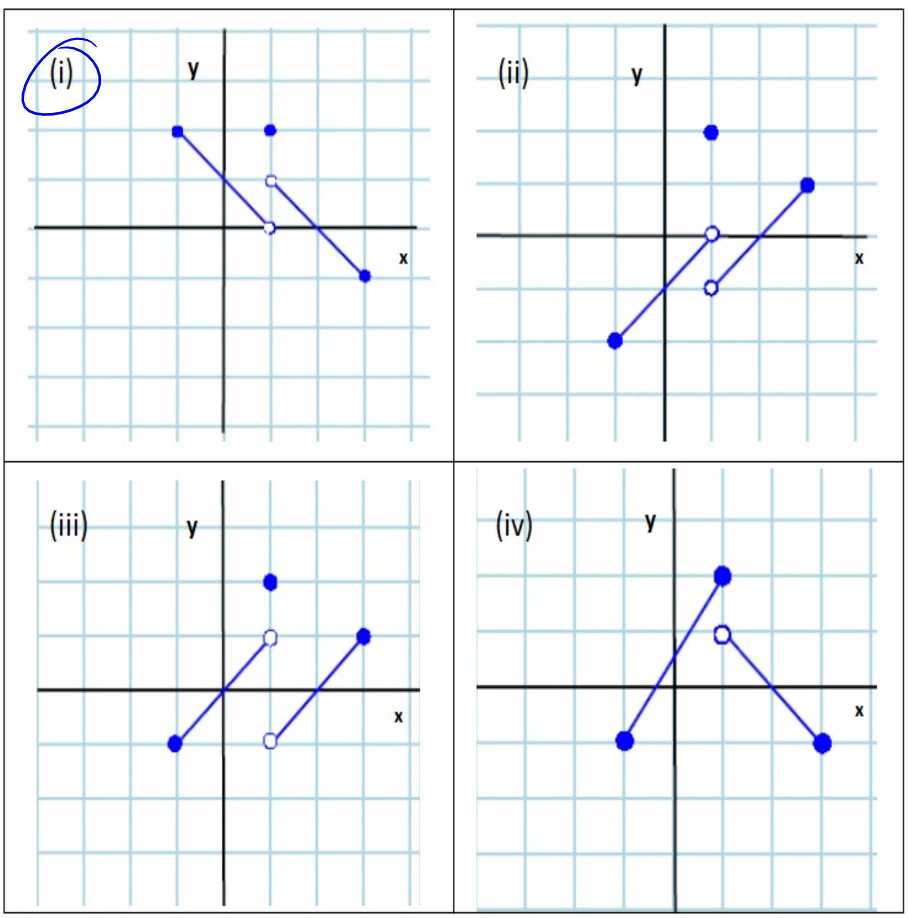
(a) Evaluate $g(1)$ $= 2$

(b) Evaluate $g\left(\frac{1}{2}\right)$ $= 1 - \frac{1}{2} = \frac{1}{2}$

(c) Evaluate $g\left(\frac{3}{2}\right)$ $= 2 - \frac{3}{2} = \frac{1}{2}$

(d) Is g a one to one function? (Justify) No, note that $g(\frac{1}{2}) = g(\frac{3}{2})$ but $\frac{1}{2} \neq \frac{3}{2}$

(e) Which of the following plots shows a graph of $y = g(x)$?



3. Each function is one to one. Find it's inverse function.

(a) $f(x) = \frac{1}{3}\ln(x-1)$

$y = \frac{1}{3}\ln(x-1)$

$x = \frac{1}{3}\ln(y-1)$

$3x = \ln(y-1)$

$e^{3x} = y-1$

$y = e^{3x}+1$

$f^{-1}(x) = e^{3x}+1$

(b) $g(x) = \dfrac{2x}{x+3}$

$y = \dfrac{2x}{x+3}$ $\quad x = \dfrac{2y}{y+3} \Rightarrow x(y+3) = 2y$

$y(x-2) = -3x$

$y = \dfrac{-3x}{x-2}$

$g^{-1}(x) = \dfrac{-3x}{x-2}$

4. Find all solutions to each equation. Your solutions should be exact (i.e. not decimal approximations).

(a) $2^x = 6^{x-1}$

$\ln 2^x = \ln 6^{x-1} \Rightarrow x\ln 2 = (x-1)\ln 6$

$x(\ln 2 - \ln 6) = -\ln 6$

$x = \dfrac{-\ln 6}{\ln 2 - \ln 6}$

(b) $\log_4(x-3)^2 = 1$ $= \log_4(4) \Rightarrow$

$(x-3)^2 = 4$

$x-3 = \pm\sqrt{4} = \pm 2$

$x = 3 \pm 2 \quad x = 5$ or $x = 1$

Check:

$\log_4(5-3)^2 = \log_4 2^2 = \log_4 4$

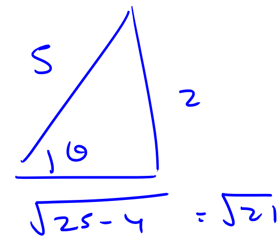
$\log_4(1-3)^2 = \log_4(-2)^2 = \log_4 4$

There are 2 answers $x=5$ or $x=1$.

5. Suppose θ is an acute angle and $\sin\theta = \frac{2}{5}$. Determine each of the other five trignometric values of θ. (Rationalizing denominators is NOT necessary.)

$\cos\theta = \frac{\sqrt{21}}{5}$ $\quad$ $\tan\theta = \frac{2}{\sqrt{21}}$ $\quad$ $\csc\theta = \frac{5}{2}$

$\sec\theta = \frac{5}{\sqrt{21}}$ $\quad$ $\cot\theta = \frac{\sqrt{21}}{2}$



6. Fill in the table with the remaining trigonometric values of the indicated angles.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$30°$	$\frac{1}{2}$	$\sqrt{3}/2$	$\frac{1}{\sqrt{3}}$
$45°$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

7. Express each as a single logarithm.

(a) $\frac{1}{3}\log_2(x+1) - \log_2(x-1) + \log_2(y)$

$$= \log_2 \sqrt[3]{x+1} - \log_2(x-1) + \log_2 y$$

$$= \log_2\left(\frac{\sqrt[3]{x+1}\; y}{x-1}\right)$$

(b) $\ln(2x) + \ln(2y) - \frac{1}{2}\ln(x^2+y^2)$ $= \ln(2x\cdot 2y) - \ln\sqrt{x^2+y^2}$

$$= \ln\left(\frac{4xy}{\sqrt{x^2+y^2}}\right)$$

8. Expand each expression as much as possible into a sum, difference, and multiple of logarithms.

(a) $\log\sqrt{\dfrac{x^3y^5}{z+1}}$ $= \frac{1}{2}\log\left(\frac{x^3y^5}{z+1}\right) = \frac{1}{2}\left(\log x^3 + \log y^5 - \log(z+1)\right)$

$= \frac{3}{2}\log x + \frac{5}{2}\log y - \frac{1}{2}\log(z+1)$

(b) $\ln\left(\dfrac{2^x}{x^3(x+8)}\right)$ $= \ln 2^x - \ln\left(x^3(x+8)\right)$

$= x\ln 2 - \left(\ln x^3 + \ln(x+8)\right)$

$= x\ln 2 - 3\ln x - \ln(x+8)$

9. Evaluate each expression exactly.

(a) $e^{\ln 7}$ $= 7$

(b) $\log(0.01)$ $= -2$

(c) $\ln\left(\frac{1}{e^3}\right)$ $= -3$

(d) $\ln(1)$ $= 0$

(e) $3^{2\log_3(3)}$ $= 3^{\log_3 3^2} = 3^2 = 9$

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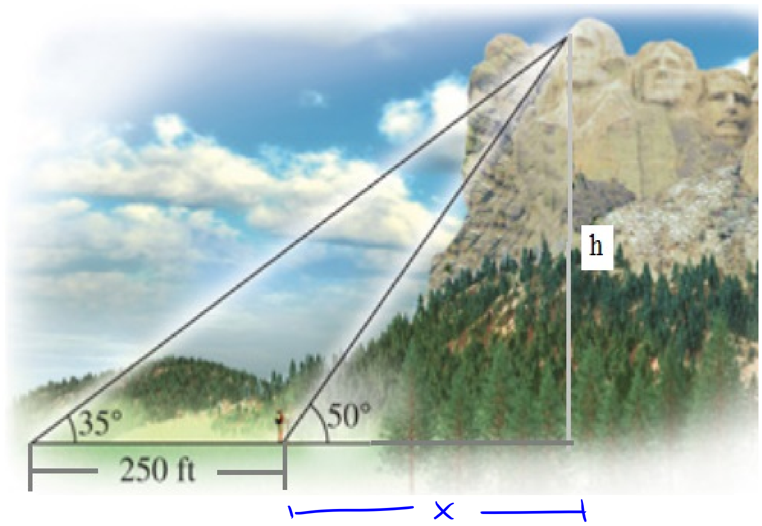
You may use a non-CAS calculator to answer this question.

10. While visiting Mount Rushmore in Rapid City, South Dakota, Landon approximated the angle of elevation to the top of George Washington's head to be $35°$. After walking 250 feet closer, he guessed that the angle of elevation had increased to $50°$. Approximate the height h of the Mount Rushmore memorial, to the top of George Washington's head. Round the answer to the nearest foot.



$$\frac{h}{x} = \tan 50° \qquad \frac{h}{x+250} = \tan 35°$$

$$h = x \tan 50° = (x+250)\tan 35°$$

$$x(\tan 50° - \tan 35°) = 250 \tan 35°$$

$$x = \frac{250 \tan 35°}{\tan 50° - \tan 35°}$$

The height

$$h = x \tan 50° = \frac{250 \tan 35° \tan 50°}{\tan 50° - \tan 35°}$$

$$\approx 424 \quad \text{feet}$$