Exam 3 Math 1113 sec. 52 Fall 2018

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

INSTRUCTIONS: There are 10 problems worth 10 points each. **No calculator use is allowed on the first 9 problems of this exam.** Once you have completed the 9 problems, turn this in and receive the last problem. There are no notes, or books allowed. **Illicit use of a smart phone, tablet, device that runs apps, or notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, and written using proper notation.

1. For the given function f, construct and simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$.

$f(x) = x^2 + 3x - 7$

$$f(x+h) = (x+h)^2 + 3(x+h) - 7$$
$$= x^2 + 2xh + h^2 + 3x + 3h - 7$$

$$\frac{f(x+h)-f(x)}{h} = \frac{x^2 + 2xh + h^2 + 3x + 3h - 7 - (x^2 + 3x - 7)}{h}$$

$$= \frac{2xh + h^2 + 3h}{h}$$

$$= \frac{h(2x + h + 3)}{h} = 2x + h + 3$$

2. Let $g(x) = \begin{cases} x-1, & -1 \le x < 1 \\ 2, & x = 1 \\ x-2, & 1 < x \le 3 \end{cases}$.

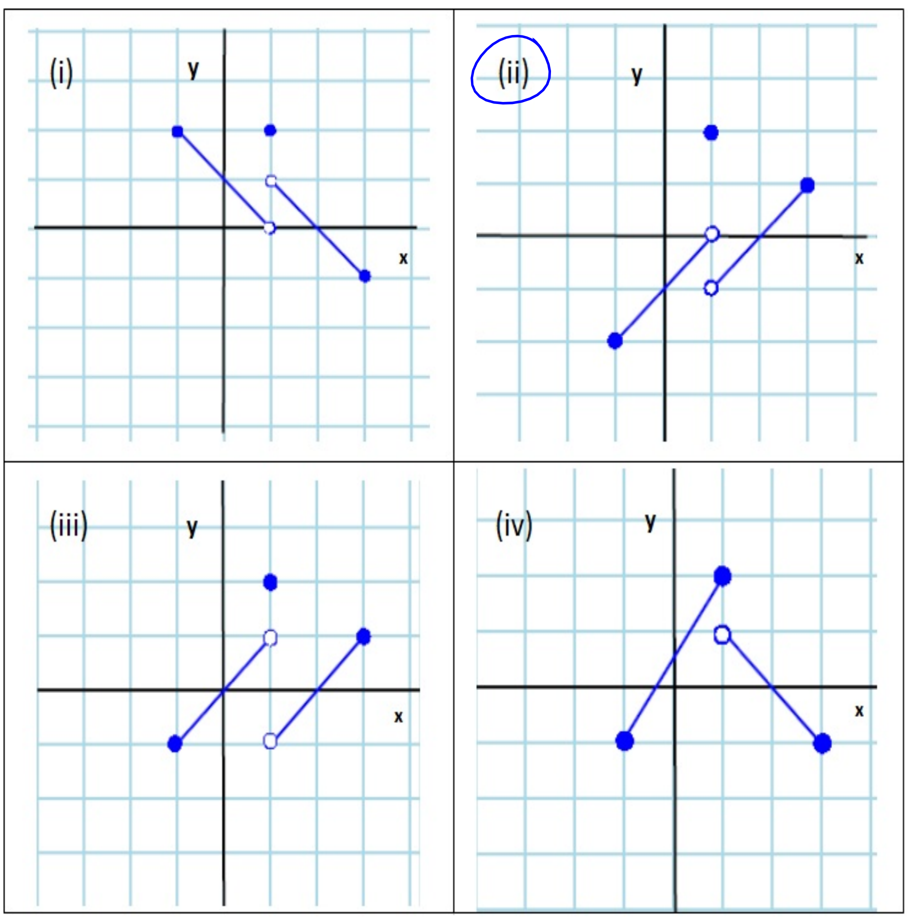
(a) Evaluate $g(1)$ $= 2$

(b) Evaluate $g\left(\frac{1}{2}\right)$ $= \frac{1}{2} - 1 = -\frac{1}{2}$

(c) Evaluate $g\left(\frac{3}{2}\right)$ $= \frac{3}{2} - 2 = -\frac{1}{2}$

(d) Is g a one to one function? (Justify) No, in fact $g\left(\frac{1}{2}\right) = g\left(\frac{3}{2}\right)$ but $\frac{1}{2} \neq \frac{3}{2}$

(e) Which of the following plots shows a graph of $y = g(x)$?



3. Each function is one to one. Find it's inverse function.

(a) $g(x) = \dfrac{3x}{x+2}$

$y = \dfrac{3x}{x+2}$

$x = \dfrac{3y}{y+2}$

$x(y+2) = 3y$

$xy - 3y = -2x$

$y(x-3) = -2x$

$y = \dfrac{-2x}{x-3}$

$g^{-1}(x) = \dfrac{-2x}{x-3}$

(b) $f(x) = 2\ln(x+1)$

$y = 2\ln(x+1)$

$x = 2\ln(y+1)$

$\ln(y+1) = \frac{1}{2}x$

$y+1 = e^{\frac{1}{2}x}$

$y = e^{\frac{1}{2}x} - 1$

$f^{-1}(x) = e^{\frac{1}{2}x} - 1$

4. Find all solutions to each equation. Your solutions should be exact (i.e. not decimal approximations).

(a) $3^x = 7^{x-1}$

$\ln 3^x = \ln 7^{x-1}$

$x \ln 3 = (x-1)\ln 7$

$x(\ln 3 - \ln 7) = -\ln 7$

$x = \dfrac{-\ln 7}{\ln 3 - \ln 7}$

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(b) $\log_4(x+3)^2 = 1$ $= \log_4 4$ $\Rightarrow$ $(x+3)^2 = 4$

$x+3 = \pm\sqrt{4} = \pm 2$

$x = -3 \pm 2$

$x = -5$ or $x = -1$

Check

$\log_4(-5+3)^2 = \log_4(-2)^2 = \log_4(4)$

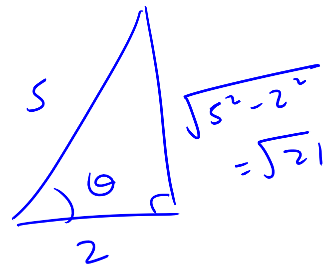
$\log_4(-1+3)^2 = \log_4(2)^2 = \log_4(4)$

Both -5 and -1 are solutions

5. Suppose θ is an acute angle and $\cos\theta = \frac{2}{5}$. Determine each of the other five trignometric values of θ. (Rationalizing denominators is NOT necessary.)

$\sin\theta = \frac{\sqrt{21}}{5}$ $\quad$ $\tan\theta = \frac{\sqrt{21}}{2}$ $\quad$ $\sec\theta = \frac{5}{2}$

$\csc\theta = \frac{5}{\sqrt{21}}$ $\quad$ $\cot\theta = \frac{2}{\sqrt{21}}$



6. Fill in the table with the remaining trigonometric values of the indicated angles.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45°$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

7. Express each as a single logarithm.

(a) $\frac{1}{4}\log_2(x-1) - \log_2(x+1) + \log_2(y)$

$$= \log_2 \sqrt[4]{x-1} - \log_2(x+1) + \log_2 y$$

$$= \log_2\left(\frac{\sqrt[4]{x-1}\; y}{x+1}\right)$$

(b) $\ln(2x) + \ln(2y) - \frac{1}{2}\ln(x^2+y^2)$ $= \ln(2x\cdot 2y) - \ln\sqrt{x^2+y^2}$

$$= \ln\left(\frac{4xy}{\sqrt{x^2+y^2}}\right)$$

8. Expand each expression as much as possible into a sum, difference, and multiple of logarithms.

(a) $\log \sqrt{\dfrac{x^5y^3}{z+2}}$ $= \frac{1}{2} \cdot \log\left(\frac{x^5y^3}{z+2}\right)$

$= \frac{1}{2}\left(\log x^5 + \log y^3 - \log(z+2)\right)$

$= \frac{5}{2} \log x + \frac{3}{2} \log y - \frac{1}{2} \log(z+2)$

(b) $\ln\left(\dfrac{4^x}{x^2(x+2)}\right)$ $= \ln 4^x - \ln\left(x^2(x+2)\right)$

$= x \ln 4 - \left(\ln x^2 + \ln(x+2)\right)$

$= x \ln 4 - 2 \ln x - \ln(x+2)$

9. Evaluate each expression exactly.

(a) $\ln\left(\frac{1}{e^4}\right)$ $= -4$

(b) $\ln(1)$ $= 0$

(c) $e^{\ln 8}$ $= 8$

(d) $\log(0.01)$ $= -2$

(e) $3^{2\log_3(3)}$ $= 3^{\log_3 3^2} = 3^2 = 9$

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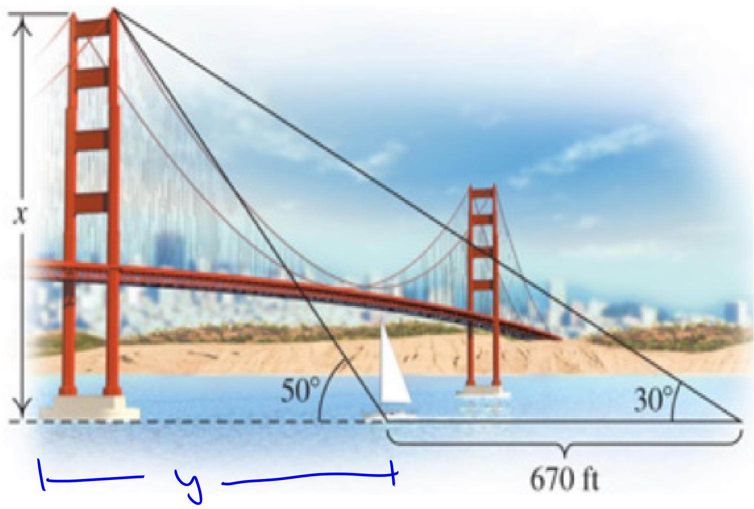
You may use a non-CAS calculator to answer this question.

10. The Golden Gate Bridge has two main towers of equal height that support the two main cables. A visitor on a tour boat passing through San Francisco Bay views the top of one of the towers and estimates the angle of elevation to be $30°$. After sailing 670 feet closer, he estimates the angle of elevation to this same tower to be $50°$. Approximate the height x of the tower to the nearest foot.



$$\frac{x}{y} = \tan 50° \qquad \frac{x}{y+670} = \tan 30°$$

$$x = y\tan 50° = (y+670)\tan 30°$$

$$y(\tan 50° - \tan 30°) = 670\tan 30°$$

$$y = \frac{670\tan 30°}{\tan 50° - \tan 30°}$$

Since $x = y\tan 50°$

$$x = \frac{670\tan 30° \tan 50°}{\tan 50° - \tan 30°} \approx 750 \text{ feet}$$