Exam 3 Math 1190 sec. 51

Fall 2016

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	

INSTRUCTIONS: There are 7 problems. The point values are listed with the problems.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.**

To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) (15 points) Find all of the critical numbers of the function whose domain is $(-\infty, \infty)$.

$f(x) = \sqrt[5]{x}(x-18)$ $= x^{6/5} - 18x^{1/5}$

$f'(x) = \frac{6}{5}x^{1/5} - 18\left(\frac{1}{5}x^{-4/5}\right) = \frac{6x^{1/5}}{5} - \frac{18}{5x^{4/5}}$

$= \frac{6x^{1/5}}{5} \cdot \frac{x^{4/5}}{x^{4/5}} - \frac{18}{5x^{4/5}} = \frac{6x - 18}{5x^{4/5}} = \frac{6(x-3)}{5x^{4/5}}$

$f'(x) = 0$ if $6(x-3) = 0$ i.e. $x = 3$

$f'(x)$ - undefined if $5x^{4/5} = 0$ i.e. $x = 0$

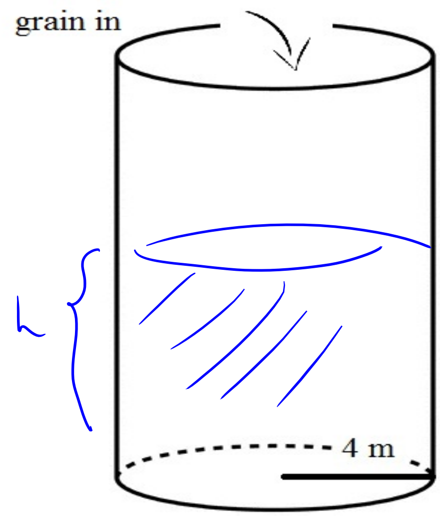
The critical number are 3 and 0.

(2) (10 points) Evaluate the limit. $\lim_{x\to 2} \frac{\ln\left(\frac{x}{2}\right)}{x^2-4}$ $= \lim_{x\to 2} \frac{\ln x - \ln 2}{(x-2)(x+2)}$ $=$ "$\frac{0}{0}$"

apply l'H's rule $= \lim_{x\to 2} \frac{\frac{1}{x}}{2x}$

$= \lim_{x\to 2} \frac{1}{2x^2} = \frac{1}{2(2)^2} = \frac{1}{8}$

(3) (15 points) A grain silo is in the shape of a tall, upright cylinder[1] of radius 4 m. Grain is being poured into the silo at the top. At the moment that the height of the grain inside the silo is 6 m, it is noticed that the height of grain is increasing at a rate of 0.5 m/min. Determine the rate of change of the volume of grain in the silo when the height is 6 m.



Let h be the height of grain in meter @ time t in minutes.

$$V = \pi r^2 h = \pi (4m)^2 h$$
$$= 16m^2 \pi h$$

so

$$\frac{d}{dt} V = \frac{d}{dt} 16m^2 \pi h$$

$$\frac{dV}{dt} = 16m^2 \pi \frac{dh}{dt}$$

When $h = 6m$, $\frac{dh}{dt} = 0.5 \frac{m}{min}$ so

at this time

$$\frac{dV}{dt} = 16m^2 \pi \left(0.5 \frac{m}{min}\right) = 8\pi \frac{m^3}{min}.$$

The volume is increasing at a rate of $8\pi \frac{m^3}{min}$ when $h = 6m$.

[1] The volume V of a cylinder of radius r and height h is

$$V = \pi r^2 h.$$

(4) (15 points) Find the absolute maximum and absolute minimum values of the function on the indicated interval.

$$f(x) = 3x^4 + 4x^3, \quad [-2, 0]$$

$f'(x) = 12x^3 + 12x^2 = 12x^2(x+1)$ Critical numbers are 0 and -1.

$f(-2) = 3(-2)^4 + 4(-2)^3 = 3 \cdot 16 - 4 \cdot 8 = 48 - 32 = 16$

$f(-1) = 3(-1)^4 + 4(-1)^3 = 3 - 4 = -1$

$f(0) = 3(0)^4 + 4(0)^3 = 0$

The absolute maximum is $16 = f(-2)$. The absolute minimum is $-1 = f(-1)$.

(5) (15 points) The variables x and y are differentiable functions of time t. Find $\frac{dy}{dt}$ when $x = 1$, $y = 3$ and $\frac{dx}{dt} = -2$.

$$x^2y^3 = 27$$

$\frac{d}{dt} x^2y^3 = \frac{d}{dt} 27$

$2xy^3 \frac{dx}{dt} + 3x^2y^2 \frac{dy}{dt} = 0$

$3x^2y^2 \frac{dy}{dt} = -2xy^3 \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{-2xy^3}{3x^2y^2} \frac{dx}{dt} = \frac{-2y}{3x} \frac{dx}{dt}$

when $x = 1$, $y = 3$, and $\frac{dx}{dt} = -2$

$\frac{dy}{dt} = \frac{-2(3)}{3(1)}(-2) = 4$

(6) (15 points) (a) Find all of the critical numbers of $f(t) = \frac{1}{4}t^4 - \frac{4}{3}t^3 + 2t^2$.

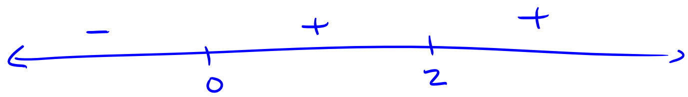
$f'(t) = t^3 - 4t^2 + 4t = t(t^2 - 4t + 4) = t(t-2)^2$

$f'(t) = 0 \Rightarrow t(t-2)^2 = 0 \Rightarrow t = 0$ or $t = 2$

$f''(t)$ is never undefined.

The critical number are 0 and 2.

(b) Determine the intervals on which f is increasing and the intervals on which f is decreasing.



sign of $f'(t)$

$f'(-1) = (-1)(-1-2)^2 = -9$, $f'(1) = 1 \cdot (1-2)^2 = 1$

$f'(3) = 3(3-2)^2 = 3$

f is increasing on $(0,2) \cup (2,\infty)$

f is decreasing on $(-\infty, 0)$.

(c) Classify each critical number as corresponding to a local maximum, a local minimum or neither.

f takes neither a local max nor min @ $x = 2$.

f takes a local minimum @ $x = 0$ by the 1st derivative test.

(7) (15 points) The polynomial function g is found to have second derivative

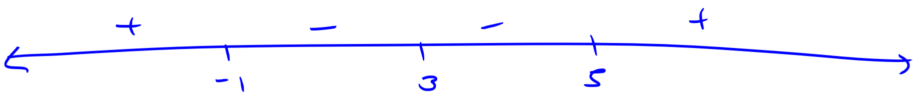
$$g''(x) = (x-3)^2(x+1)(x-5).$$

Determine the intervals on which the graph of g is concave up, the intervals on which it is concave down, and determine all x values at which the graph of g has a point of inflection.

$g''(x) = 0$ if $(x-3)^2(x+1)(x-5) = 0$

$x = 3, -1,$ or 5

$g''(x)$ is always well defined.



sign of $g''(x)$

$g''(-2) = (-2-3)^2(-2+1)(-2-5)$
$\quad + \quad - \quad -$

$g''(4) = (4-3)^2(4+1)(4-5)$
$\quad + \quad + \quad -$

$g''(0) = (0-3)^2(0+1)(0-5)$
$\quad + \quad + \quad -$

$g''(6) = (6-3)^2(6+1)(6-5)$
$\quad + \quad + \quad +$

g is concave up on $(-\infty, -1) \cup (5, \infty)$.

g is concave down on $(-1, 3) \cup (3, 5)$.

g has inflection points @ $(-1, g(-1))$ and at $(5, g(5))$ as concavity changes here.