Exam 3 Math 1190 sec. 51

Summer 2017

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	

INSTRUCTIONS: You have 60 minutes to complete this exam.

There are 8 problems. The point values are listed with the problems.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) (15 points, 5 each) Evaluate each limit using any applicable technique.

a) $\lim_{x \to 0} \frac{\sin(3x)}{x}$ $= \lim_{x \to 0} 3 \frac{\sin(3x)}{3x} = 3 \cdot 1 = 3$

b) $\lim_{t \to 0} \frac{e^{-t} - 1}{2t}$ $=$ " $\frac{1-1}{0}$ " $=$ " $\frac{0}{0}$ "

l'H rule $= \lim_{t \to 0} \frac{-e^{-t}}{2} = \frac{-e^{0}}{2} = \frac{-1}{2}$

c) $\lim_{x \to 1} \frac{x^2 + 1}{x^2 - 2}$ $= \frac{1+1}{1-2} = \frac{2}{-1} = -2$

(2) (10 points) Find the absolute maximum value and the absolute minimum value of the function on the given interval.

$$f(x) = x^3 - 3x + 4, \quad \text{on} \quad [0, 3]$$

$f'(x) = 3x^2 - 3 = 3(x^2-1) = 3(x-1)(x+1)$

$f'(x)$ always exists

$f'(x) = 0 \Rightarrow x = 1$ or $x = -1$

Only the crit. # 1 is on the interval.

$f(0) = 0^3 - 3 \cdot 0 + 4 = 4$

$f(1) = 1^3 - 3 \cdot 1 + 4 = 1 - 3 + 4 = 2$ ← min

$f(3) = 3^3 - 3 \cdot 3 + 4 = 27 - 9 + 4 = 22$ ← max

The abs. max is $22 = f(3)$. The abs min is $2 = f(1)$.

(3) (10 points) Suppose g is continuous on $(-\infty, \infty)$ with second derivative

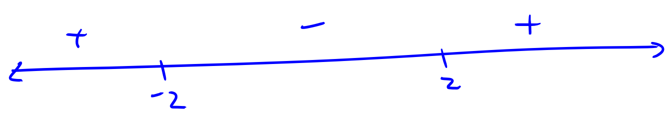
$$g''(x) = e^x(x^2 - 4). = e^x (x-2)(x+2)$$

Determine the intervals over which the graph of g is concave up and the intervals over which it is concave down. Identify the x-value(s), if any, at which g has a point of inflection.

$g''(x)$ is always defined

$g''(x) = 0 \Rightarrow e^x = 0$ or $x - 2 = 0$ or $x + 2 = 0$ $\Rightarrow x = 2$ or $x = -2$

Sign of g''



Test
$g''(-3) = e^{-3}(-3-2)(-3+2)$ $+ \; - \; -$
$g''(0) = e^{0}(0-2)(0+2)$ $+ \; - \; +$
$g''(3) = e^{3}(3-2)(3+2)$ $+ \; + \; +$

g is concave up on $(-\infty, -2) \cup (2, \infty)$ g is concave down on $(-2, 2)$

g has inflection point(s) at -2 and 2 (Write "none" if there are no inflection points.)

(4) (10 points, 5 each) Find all critical numbers of each function.

a) $f(x) = x^4 - \frac{8}{3}x^3 + 2x^2 + 1$ Domain - all reals

$f'(x) = 4x^3 - 8x^2 + 4x = 4x(x^2 - 2x + 1) = 4x(x-1)^2$

$f'(x)$ exist everywhere

$f'(x) = 0 \Rightarrow 4x = 0$ or $(x-1)^2 = 0$

$x = 0$ or $x = 1$

The crit #s are 0 and 1.

b) $f(x) = \dfrac{x+1}{x^2+3}$

Domain all reals

$f'(x) = \dfrac{1(x^2+3) - 2x(x+1)}{(x^2+3)^2} = \dfrac{x^2+3-2x^2-2x}{(x^2+3)^2}$

$= \dfrac{-x^2-2x+3}{(x^2+3)^2}$

$f''(x)$ DNE if $x^2+3=0$
no solutions

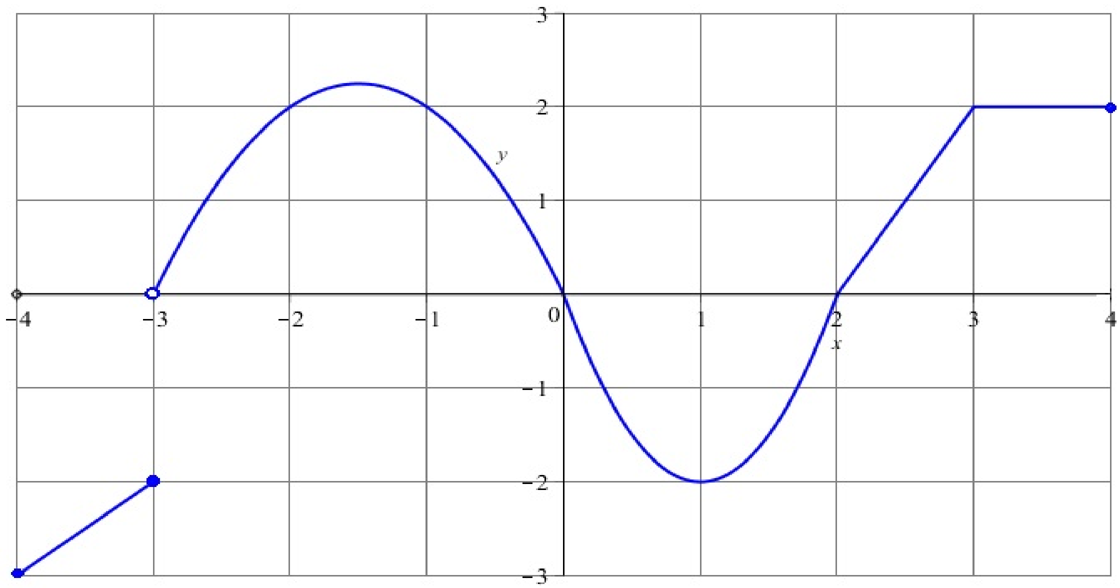
$f''(x) = 0 \Rightarrow -x^2 - 2x + 3 = 0 \Rightarrow -(x^2+2x-3) = 0$

$-(x+3)(x-1) = 0$

$x = -3$ or $x = 1$

The two crit #'s are -3 and 1.

(5) Use the graph of $y = f(x)$ shown to evaluate or answer each question.



a) (3 points) Is $f''(-2)$ positive, negative, or zero? negative (concave down)

b) (3 points) Evaluate (or write DNE) $f'(3) =$ DNE (corner)

c) (3 points) Evaluate (or write DNE) $f'(1) =$ 0 (horizontal tangent)

d) (2 points) The graph of f has one point of inflection. Which of the following could be its location? (circle one)

$$x = -2, \quad x = -1, \quad x = 0, \quad x = 1$$

Concavity change

e) (2 points) Which value is greater $f'\left(\frac{1}{2}\right)$ or $f''\left(\frac{1}{2}\right)$? $f''(\frac{1}{2}) > f'(\frac{1}{2})$

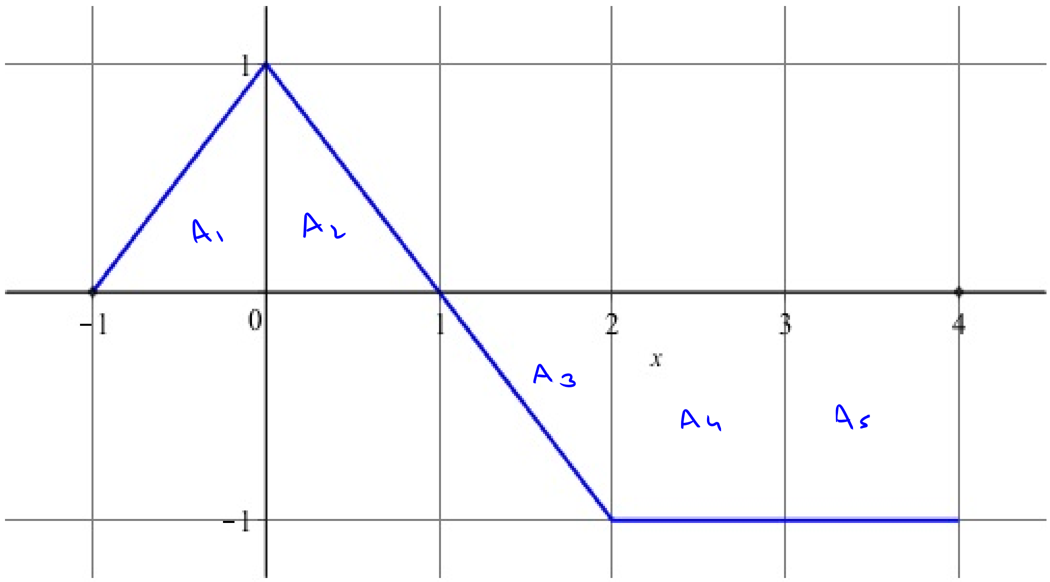
neg. slope pos. concave up

f) (2 points) Jack claims that $\int_0^2 f(x)\,dx$ must be positive because the graph there is concave up. Diane claims that $\int_0^2 f(x)\,dx$ must be negative because the graph there is below the x-axis. One of them is correct. Who is correct? Why?

Diane is correct. The region is below the x-axis. Hence $\int_0^2 f(x)\,dx = -$ Area

Concavity is n't relevant.

(6) The figure shows the graph of $y = f(x)$.



$A_1 = A_2 = A_3 = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$

$A_4 = A_5 = 1 \cdot 1 = 1$

(12 points, 4 each) Use the graph to evaluate each integral.

(a) $\int_{-1}^{1} f(x)\,dx = A_1 + A_2 = 1$

(b) $\int_{-1}^{2} f(x)\,dx = A_1 + A_2 - A_3 = \frac{1}{2}$

(c) $\int_{0}^{4} f(x)\,dx = A_2 - A_3 - A_4 - A_5 = \frac{1}{2} - \frac{1}{2} - 1 - 1 = -2$

(7) (20 points, 4 each) Determine the **most general** antiderivative of each function. (Use the uppercase/lowercase convention for naming your antiderivatives.)

a) $f(x) = 2x - 3$

$F(x) = x^2 - 3x + C$

b) $g(t) = \cos t - \sin t$

$G(t) = \sin t + \cos t + C$

c) $f(x) = \dfrac{1}{1+x^2}$

$F(x) = \tan^{-1} x + C$

d) $h(x) = \dfrac{4x^4 + 3x^3}{x} = 4x^3 + 3x^2$

$H(x) = x^4 + x^3 + C$

e) $y = 3e^{3x}$

$Y = e^{3x} + C$

(8) (8 points) Find $\dfrac{dy}{dx}$ given $y = x^x$.

$\ln y = \ln x^x = x \ln x$

$\frac{d}{dx} \ln y = \frac{d}{dx} x \ln x$

$\frac{1}{y} \frac{dy}{dx} = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$

$\frac{dy}{dx} = y(\ln x + 1) \Rightarrow \boxed{\frac{dy}{dx} = x^x(\ln x + 1)}$