Exam 3 Math 1190 sec. 52

Fall 2016

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ____________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	

INSTRUCTIONS: There are 7 problems. The point values are listed with the problems.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.**

To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) (15 points) Find all of the critical numbers of the function whose domain is $(-\infty, \infty)$.

$f(x) = \sqrt[5]{x}(x-24) = x^{6/5} - 24x^{1/5}$

$$f'(x) = \frac{6}{5}x^{1/5} - 24\left(\frac{1}{5}x^{-4/5}\right) = \frac{6x^{1/5}}{5} - \frac{24}{5x^{4/5}}$$

$$= \frac{6x^{1/5}}{5} \cdot \frac{x^{4/5}}{x^{4/5}} - \frac{24}{5x^{4/5}} = \frac{6x - 24}{5x^{4/5}} = \frac{6(x-4)}{5x^{4/5}}$$

$f'(x) = 0 \Rightarrow 6(x-4) = 0 \Rightarrow x = 4$

$f'(x)$ is undefined if $5x^{4/5} = 0 \Rightarrow x = 0$.

$\boxed{\text{The critical numbers are 4 and 0.}}$

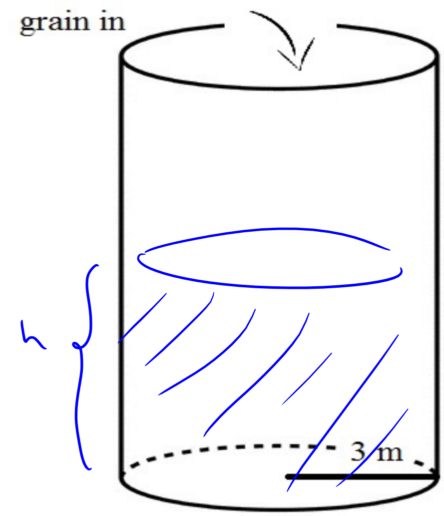
(2) (10 points) Evaluate the limit. $\displaystyle\lim_{x\to 4} \frac{x^2-16}{\ln\left(\frac{x}{4}\right)} = \lim_{x\to 4} \frac{x^2-16}{\ln x - \ln 4} = \text{"}\frac{0}{0}\text{"}$

apply l'H's rule $\displaystyle = \lim_{x\to 4} \frac{2x}{\frac{1}{x}}$

$$= \lim_{x\to 4} 2x^2 = 2(4)^2 = 32$$

(3) (15 points) A grain silo is in the shape of a tall, upright cylinder[1] of radius 3 m. Grain is being poured into the silo at the top. At the moment that the height of the grain inside the silo is 6 m, it is noticed that the height of grain is increasing at a rate of $\frac{1}{3}$ m/min. Determine the rate of change of the volume of grain in the silo when the height is 6 m.

Let h be the height of the grain in meters @ time t in minutes.



Volume $V = \pi r^2 h$

$= \pi (3m)^2 h$

$= 6m^2 \pi h$

$$\frac{d}{dt} V = \frac{d}{dt}(6m^2 \pi h)$$

$$\frac{dV}{dt} = 6m^2 \pi \frac{dh}{dt}$$

When $h = 6m$, $\frac{dh}{dt} = \frac{1}{3} \frac{m}{min}$ so that

$$\frac{dV}{dt} = 6m^2 \pi \left(\frac{1}{3} \frac{m}{min}\right) = 2\pi \frac{m^3}{min}.$$

At this time, the volume is increasing @ 2π m^3/min.

[1]The volume V of a cylinder of radius r and height h is

$$V = \pi r^2 h.$$

(4) (15 points) Find the absolute maximum and absolute minimum values of the function on the indicated interval.

$$f(x) = 3x^4 - 4x^3, \quad [0, 2]$$

$f'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$. Critical number 0, 1.

$f(0) = 3 \cdot 0^4 - 4 \cdot 0^3 = 0$

$f(1) = 3 \cdot 1^4 - 4(1)^3 = 3 - 4 = -1$

$f(2) = 3(2)^4 - 4(2)^3 = 48 - 32 = 16$

The absolute maximum is $16 = f(2)$. The absolute minimum is $-1 = f(1)$.

(5) (15 points) The variables x and y are differentiable functions of time t. Find $\frac{dy}{dt}$ when $x = 1$, $y = 2$ and $\frac{dx}{dt} = -3$.

$$x^2y^3 = 8$$

$\frac{d}{dt} x^2y^3 = \frac{d}{dt} 8$

$2xy^3 \frac{dx}{dt} + 3x^2y^2 \frac{dy}{dt} = 0$

$3x^2y^2 \frac{dy}{dt} = -2xy^3 \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{-2xy^3}{3x^2y^2} \frac{dx}{dt} = \frac{-2y}{3x} \frac{dx}{dt}$

When $x = 1$, $y = 2$ and $\frac{dx}{dt} = -3$

$\frac{dy}{dt} = \frac{-2 \cdot 2}{3 \cdot 1} (-3) = 4$

(6) (15 points) (a) Find all of the critical numbers of $f(t) = \frac{1}{4}t^4 - 2t^3 + \frac{9}{2}t^2$.

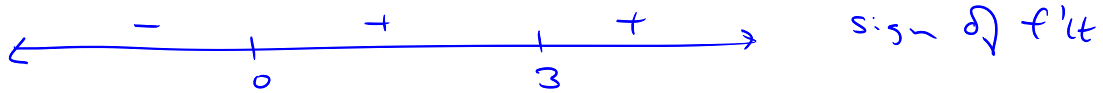
$$f'(t) = t^3 - 6t^2 + 9t = t(t^2 - 6t + 9) = t(t-3)^2$$

$$f'(t) = 0 \Rightarrow t(t-3)^2 = 0 \Rightarrow t = 0 \text{ or } t = 3.$$

$f'(t)$ is never undefined.

The critical numbers are 0 and 3.

(b) Determine the intervals on which f is increasing and the intervals on which f is decreasing.



$$f'(-1) = -1(-1-3)^2 = -16 \qquad f'(4) = 4(4-3)^2 = 4$$

$$f'(1) = 1(1-3)^2 = 4$$

f is increasing on $(0,3) \cup (3,\infty)$.

f is decreasing on $(-\infty, 0)$.

(c) Classify each critical number as corresponding to a local maximum, a local minimum or neither.

f takes a local minimum @ $(0, f(0))$.

f takes neither a max or min @ $(3, f(3))$.

by the 1st derivative test.

(7) (15 points) The polynomial function g is found to have second derivative

$$g''(x) = (x+2)^2(x-1)(x-4).$$

Determine the intervals on which the graph of g is concave up, the intervals on which it is concave down, and determine all x values at which the graph of g has a point of inflection.

$g''(x) = 0 \Rightarrow x = -2,\ x = 1$ or $x = 4$

$g''(x)$ is always defined.

$$\begin{array}{ccccccccc} & + & & + & & - & & + & \\ \longleftarrow & & \big| & & \big| & & \big| & & \longrightarrow \\ & & -2 & & 1 & & 4 & & \end{array}$$

Sign of $g''(x)$

$$g''(-3) = \underset{+}{(-3+2)^2}\underset{-}{(-3-1)}\underset{-}{(-3-4)}$$

$$g''(5) = \underset{+}{(5+2)^2}\underset{+}{(5-1)}\underset{+}{(5-4)}$$

$$g''(0) = \underset{+}{(0+2)^2}\underset{-}{(0-1)}\underset{-}{(0-4)}$$

$$g''(2) = \underset{+}{(2+2)^2}\underset{+}{(2-1)}\underset{-}{(2-4)}$$

g's graph is concave up on $(-\infty, -2) \cup (-2, 1) \cup (4, \infty)$.

it's concave down on $(1, 4)$.

g has inflection points @ $x = 1$ and 4

where the concavity changes.