Exam 3 Math 1190 sec. 62

Spring 2017

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ____________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	

INSTRUCTIONS: You have 50 minutes to complete this exam.

There are 7 problems. The point values are listed with the problems.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) (18 points) Evaluate each limit using any applicable technique.

(a) $\lim_{x\to 0} \dfrac{2^x - 1}{x}$ $=$ "$\frac{1-1}{0}$" $=$ "$\frac{0}{0}$" use l'H rule

$= \lim_{x\to 0} \dfrac{2^x \ln 2}{1} = 2^0 \ln 2 = \ln 2$

(b) $\lim_{x\to 1} \dfrac{x^2 - 1}{x^2 + 1}$ $= \dfrac{1^2-1}{1^2+1} = \dfrac{0}{2} = 0$

(c) $\lim_{x\to 0^+} (1 + 2x)^{1/x}$ $=$ "1^∞"

Note $\ln (1+2x)^{\frac{1}{x}} = \frac{1}{x} \ln(1+2x) = \dfrac{\ln(1+2x)}{x}$

$\lim_{x\to 0^+} \dfrac{\ln(1+2x)}{x} =$ "$\frac{0}{0}$" use l'H rule

$= \lim_{x\to 0^+} \dfrac{\frac{2}{1+2x}}{1} = \dfrac{2}{1+0} = 2$

So $\lim_{x\to 0^+} (1+2x)^{\frac{1}{x}} = e^2$

(2) (a) (8 points) Find all of the critical numbers of the function $f(x) = 2x^3 + 3x^2$.

$f'(x) = 6x^2 + 6x = 6x(x+1)$, $f'(x)$ is always defined

$f'(x) = 0 \Rightarrow 6x(x+1) = 0$

$x = 0$ or $x = -1$

The two crit. #'s are 0 and -1.

(b) (8 points) Find the absolute maximum and absolute minimum values of $f(x) = 2x^3 + 3x^2$ on the interval $[-2, 1]$.

Check the ends and the critical numbers.

$f(-2) = 2(-8) + 3 \cdot 4 = -4$

$f(-1) = 2(-1) + 3(1) = 1$

$f(0) = 0$

$f(1) = 2 + 3 = 5$

The absolute max value is

$5 = f(1)$

The absolute min value is

$-4 = f(-2)$.

(3) (12 points) Find the most general antiderivative of each function.

(a) $y(x) = \dfrac{x+1}{x}$ $= 1 + \frac{1}{x}$ $Y(x) = x + \ln|x| + C$

(b) $f(x) = 4x^3 + e^x$ $F(x) = x^4 + e^x + C$

(c) $g(x) = \dfrac{1}{\sqrt{1 - x^2}}$ $G(x) = \sin^{-1} x + C$

(4) (18 points) Find $\frac{dy}{dx}$ using any applicable technique.

(a) $y = \log_2(x^2+3)$

$$y' = \frac{2x}{(x^2+3)\ln 2}$$

(b) $y = x^{\cos x}$

$$\ln y = \ln x^{\cos x} = \cos x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = -\sin x \ln x + \cos x \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = y\left(\frac{\cos x}{x} - \sin x \ln x\right) = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \ln x\right)$$

(c) $y = \ln\left(\frac{x}{x+1}\right) = \ln x - \ln(x+1)$

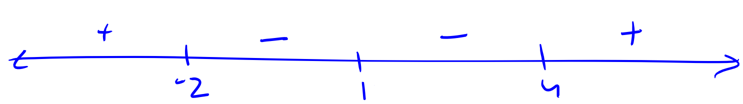
$$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x+1}$$

(5) (12 points) Suppose f is continuous on $(-\infty, \infty)$ and has the derivative

$$f'(x) = (x-1)^2(x+2)(x-4).$$

Determine the intervals on which f is increasing and the intervals on which it is decreasing.

$f'(x)$ is always defined $\quad f'(x) = 0 \Rightarrow x = 1,\ x = -2$ or $x = 4$



sign of f'

test

$f'(-3)$ $(+)(-)(-)$ $\quad$ $f'(2)$ $(+)(+)(-)$

$f'(-1)$ $(+)(+)(-)$ $\quad$ $f'(5)$ $(+)(+)(+)$

f is increasing on $(-\infty, -2) \cup (4, \infty)$ $\quad$ f is decreasing on $(-2, 1) \cup (1, 4)$

(6) (14 points) Determine the interval(s) over which $f(x) = xe^{-x}$ is concave up, and the interval(s) over which it is concave down. Identify all x-values (if there are any) at which f has a point of inflection.

$$f'(x) = 1 \cdot e^{-x} + x \cdot e^{-x}(-1) = e^{-x}(1-x)$$

$$f''(x) = e^{-x}(-1) + (1-x)e^{-x}(-1) = -e^{-x} - e^{-x} + xe^{-x}$$

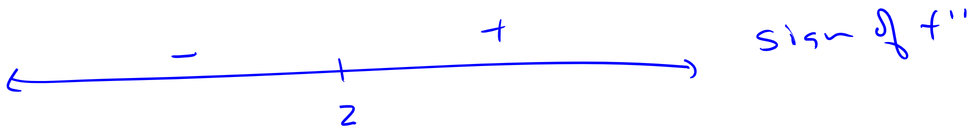
$$= e^{-x}(x-2)$$

$f''(x)$ is always defined.

$$f''(x) = 0 \Rightarrow e^{-x}(x-2) = 0$$

$e^{-x} = 0$ or $x - 2 = 0$

no soln. $\quad x = 2$



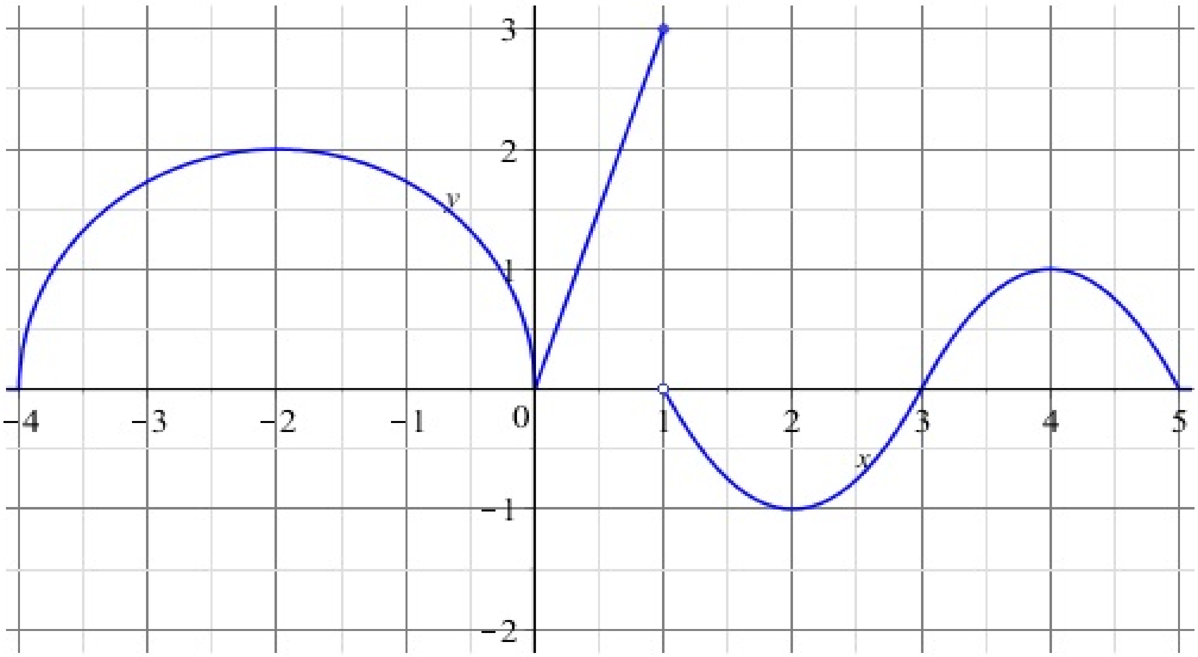
test $f''(0) = e^{-0}(0-2)$ $(+)(-)$

$f''(3) = e^{-3}(3-2)$ $(+)(+)$

f is concave up on $(2, \infty)$ f is concave down on $(-\infty, 2)$

f has point(s) of inflection at $x = 2$ (write "none" if there are none)

(7) (10 points) Use the graph of $y = f(x)$ shown to evaluate or answer the following questions.



a. Evaluate if possible (otherwise write DNE) $f'(4) =$ 0 (horizontal tangent)

b. Is $f''(x)$ positive, negative, or zero on the interval $1 < x < 3$? (How do you know?)

Positive, graph is concave up

c. f has one point of inflection on the interval $[-4, 5]$. Which of the following could be its location: (circle one)

$x = -2$ $\quad$ $x = 1$ $\quad$ $x = 2$ $\quad$ $x = 3$ $\quad$ $x = 4$

d. Evaluate if possible (otherwise write DNE) $\lim_{x \to 1^+} f(x) =$ 0

e. Which number is greater $f'(-3)$ or $f''(-3)$? (How do you know?)

$f'(-3) > 0$ sloped up

$f''(-3) < 0$ concave down

$\Rightarrow$ $f'(-3) > f''(-3)$