Exam 3 Math 1190 sec. 63

Spring 2017

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	

INSTRUCTIONS: You have 50 minutes to complete this exam.

There are 7 problems. The point values are listed with the problems.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) (18 points) Evaluate each limit using any applicable technique.

(a) $\lim_{x\to 0} \frac{1-3^x}{x}$ $=$ "$\frac{1-1}{0}$" $=$ "$\frac{0}{0}$" use l'H rule

$= \lim_{x\to 0} \frac{-3^x \ln 3}{1} = -3^0 \ln 3 = -\ln 3$

(b) $\lim_{x\to 2} \frac{x^2-4}{x^2+4}$ $= \frac{2^2-4}{2^2+4} = \frac{0}{8} = 0$

(c) $\lim_{x\to 0^+} (1+4x)^{1/x}$ $=$ "1^∞"

Note $\ln (1+4x)^{\frac{1}{x}} = \frac{1}{x} \ln (1+4x) = \frac{\ln(1+4x)}{x}$

$\lim_{x\to 0^+} \frac{\ln(1+4x)}{x} =$ "$\frac{0}{0}$" use l'H rule

$= \lim_{x\to 0^+} \frac{\frac{4}{1+4x}}{1} = \frac{4}{1+0} = 4$

Hence $\lim_{x\to 0^+} (1+4x)^{\frac{1}{x}} = e^4$

(2) (a) (8 points) Find all of the critical numbers of the function $f(x) = 3x^2 - 2x^3$.

$f'(x) = 6x - 6x^2 = 6x(1-x)$, f' is never undefined

$f'(x) = 0 \Rightarrow 6x(1-x) = 0 \Rightarrow x = 0$ or $x = 1$

The two critical numbers are 0 and 1.

(b) (8 points) Find the absolute maximum and absolute minimum values of $f(x) = 3x^2 - 2x^3$ on the interval $[-1, 2]$.

Check f @ end pts and critical numbers.

$f(-1) = 3 - 2(-1) = 5$

$f(0) = 0$

$f(1) = 3 - 2 = 1$

$f(2) = 3 \cdot 4 - 2 \cdot 8 = -4$

The absolute max value is

$5 = f(-1)$.

The absolute min value is

$-4 = f(2)$.

(3) (12 points) Find the most general antiderivative of each function.

(a) $y(x) = \dfrac{2x^2 - 1}{x} = 2x - \dfrac{1}{x}$ $\qquad Y(x) = x^2 - \ln|x| + C$

(b) $f(x) = 3e^{3x} - 5x^4$ $\qquad F(x) = e^{3x} - x^5 + C$

(c) $g(x) = \dfrac{1}{1 + x^2}$ $\qquad G(x) = \tan^{-1}x + C$

(4) (18 points) Find $\frac{dy}{dx}$ using any applicable technique.

(a) $y = \csc(\ln x)$

$$y' = -\csc(\ln x)\cot(\ln x)\cdot\frac{1}{x}$$

(b) $y = (\tan x)^x$

$$\ln y = \ln(\tan x)^x = x\ln(\tan x)$$

$$\frac{1}{y}\frac{dy}{dx} = \ln(\tan x) + x\,\frac{\sec^2 x}{\tan x}$$

$$\frac{dy}{dx} = y\left(\ln \tan x + \frac{x\sec^2 x}{\tan x}\right) = (\tan x)^x\left(\ln(\tan x) + x\,\frac{\sec^2 x}{\tan x}\right)$$

(c) $y = \log_4(x^3 + 1)$

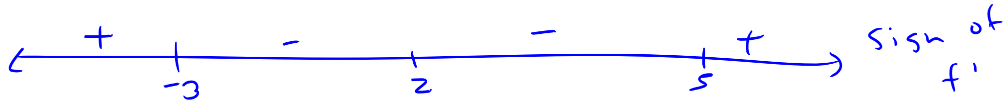
$$y' = \frac{3x^2}{(x^3+1)\ln 4}$$

(5) (12 points) Suppose f is continuous on $(-\infty, \infty)$ and has the derivative

$$f'(x) = (x+3)(x-2)^2(x-5).$$

Determine the intervals on which f is increasing and the intervals on which it is decreasing.

f' is always defined $\quad f'(x) = 0 \Rightarrow x = -3, 2, 5$



Test

$f'(-4)$ $(-)(+)(-)$ $\quad$ $f'(3)$ $(+)(+)(-)$

$f'(0)$ $(+)(+)(-)$ $\quad$ $f'(6)$ $(+)(+)(+)$

f is increasing on $(-\infty, -3) \cup (5, \infty)$ $\quad$ f is decreasing on $(-3, 2) \cup (2, 5)$

(6) (14 points) Determine the interval(s) over which $f(x) = xe^{-x}$ is concave up, and the interval(s) over which it is concave down. Identify all x-values (if there are any) at which f has a point of inflection.

$f'(x) = 1 \cdot e^{-x} + x \cdot e^{-x}(-1) = e^{-x}(1-x)$

$f''(x) = e^{-x}(-1) + (1-x)e^{-x}(-1) = -e^{-x} - e^{-x} + xe^{-x}$

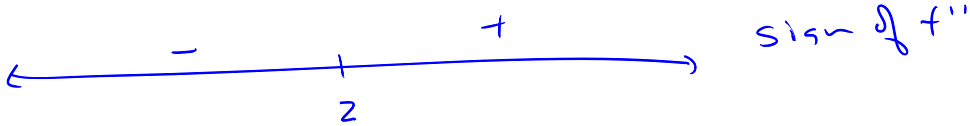
$= e^{-x}(x-2)$

$f''(x)$ is always defined.

$f''(x) = 0 \Rightarrow e^{-x}(x-2) = 0$

$e^{-x} = 0$ or $x - 2 = 0$

no soln. $\quad x = 2$



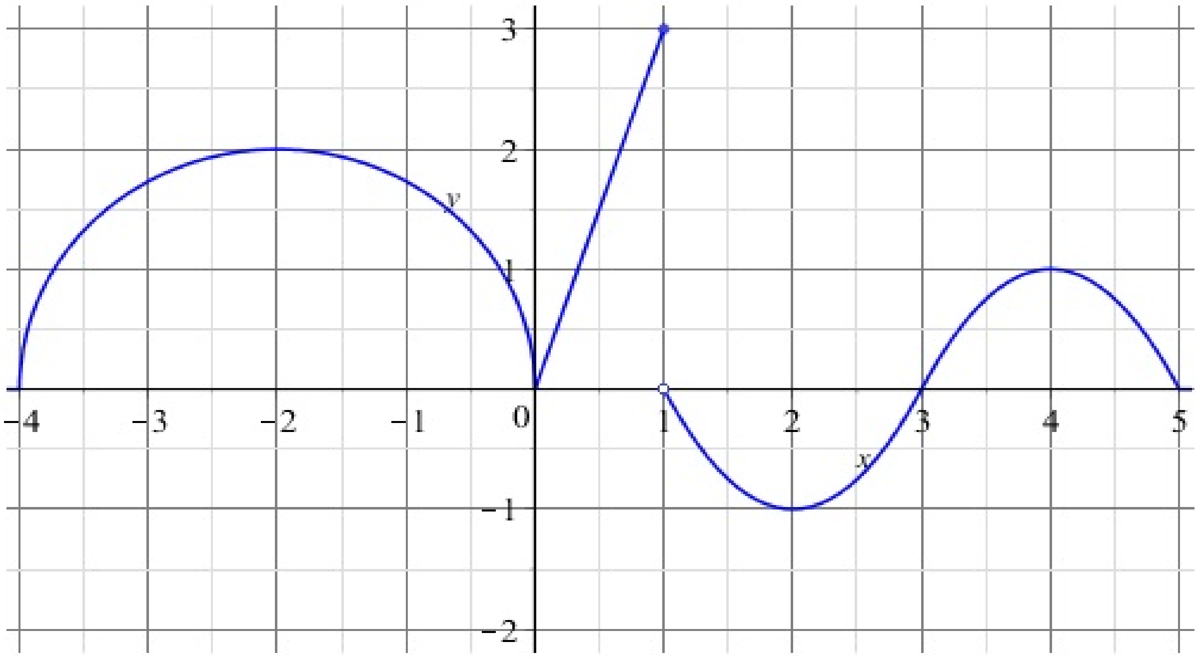
test $f''(0) = e^{-0}(0-2)$ $(+)(-)$

$f''(3) = e^{-3}(3-2)$ $(+)(+)$

f is concave up on $(2, \infty)$ $\quad$ f is concave down on $(-\infty, 2)$

f has point(s) of inflection at $x = 2$ (write "none" if there are none)

(7) (10 points) Use the graph of $y = f(x)$ shown to evaluate or answer the following questions.



a. Evaluate if possible (otherwise write DNE) $f'(2) =$ 0 (horizontal tangent)

b. Is $f''(x)$ positive, negative, or zero on the interval $3 < x < 5$? (How do you know?)

negative, graph is concave down

c. f has one point of inflection on the interval $[-4, 5]$. Which of the following could be its location: (circle one)

$x = -3$ $\quad x = -1$ $\quad x = 2$ $\quad x = 3$ $\quad x = 4$

d. Evaluate if possible (otherwise write DNE) $\lim_{x \to 1^-} f(x) =$ 3

e. Which number is greater $f'\left(\frac{3}{2}\right)$ or $f''\left(\frac{3}{2}\right)$? (How do you know?)

$$\left.\begin{array}{ll} f'(\tfrac{3}{2}) < 0 & \text{slope down} \\ f''(\tfrac{3}{2}) > 0 & \text{concave up} \end{array}\right\} \Rightarrow f''(\tfrac{3}{2}) > f'(\tfrac{3}{2}).$$