Exam 4 Math 1112 sec. 54 Spring 2019

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ____________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	
Total	

INSTRUCTIONS: There are 7 problems; point values are listed with the problems. No use of notes, books or calculators is allowed. **Illicit use of a calculator, smart phone, tablet, device that runs apps, or notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, and written using proper notation.

You may assume the following IDs.

$$\cos(u+v) = \cos u\cos v - \sin u\sin v$$

$$\sin(u+v) = \sin u\cos v + \sin v\cos u$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1-\tan u\tan v}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

1. (10 points) One of the Pythagorean Identities is $\tan^2 x + 1 = \sec^2 x$. State the other two.

One is $\sin^2 x + \cos^2 x = 1$,

the other is $\cot^2 x + 1 = \csc^2 x$

2. (12 points, 6 each) Verify each trigonometric identity. (Your steps should be clear and correct.)

(a) $\cos x + \cos x \tan^2 x = \sec x$

From the left side

$\cos x + \cos x \tan^2 x = \cos x\,(1 + \tan^2 x)$ factor

$= \cos x\,(\sec^2 x)$ Pythagorean ID

$= \cos x \sec x \sec x$

$= 1 \cdot \sec x$ reciprocal ID

$= \sec x$

(b) $\dfrac{\cot\theta}{\csc\theta - 1} = \dfrac{\csc\theta + 1}{\cot\theta}$

From the left

$\dfrac{\cot\theta}{\csc\theta - 1} = \left(\dfrac{\cot\theta}{\csc\theta - 1}\right)\left(\dfrac{\csc\theta + 1}{\csc\theta + 1}\right)$ Algebra

$= \dfrac{\cot\theta\,(\csc\theta + 1)}{\csc^2\theta - 1}$

$= \dfrac{\cot\theta\,(\csc\theta + 1)}{\cot^2\theta}$ Pythagorean ID.

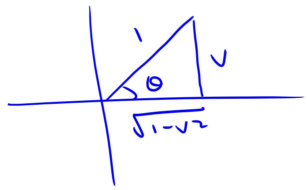
$= \dfrac{\csc\theta + 1}{\cot\theta}$ algebra

3. (8 points) Find an algebraic expression equal to $\tan\left(\sin^{-1} V\right) = \dfrac{V}{\sqrt{1-V^2}}$

Let $\theta = \sin^{-1} V$

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

and $\sin\theta = V$



4. (20 points, 5 each) Evaluate each exactly. Use appropriate sum, difference, or half angle identities that were provided. (Do not leave complex fractions in your answers; it is not necessary to rationalize.)

(a) $\cos\left(\frac{\pi}{8}\right) = \cos\left(\frac{\pi/4}{2}\right) = \sqrt{\frac{1+\cos \pi/4}{2}} = \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$

$\frac{\pi}{8}$ is acute

$\cos\frac{\pi}{8} > 0$

(b) $\sin(20°)\cos(25°) + \sin(25°)\cos(20°)$

$= \sin(20° + 25°) = \sin(45°) = \frac{1}{\sqrt{2}}$

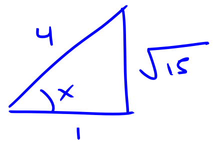
(c) $\sin(75°) = \sin(30° + 45°) = \sin 30° \cos 45° + \sin 45° \cos 30°$

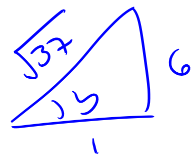
$= \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$

(d) $\dfrac{\tan(10°) + \tan(35°)}{1 - \tan(10°)\tan(35°)} = \tan(10° + 35°) = \tan(45°) = 1$

5. (20 points) Suppose x and y are **acute** angles such that $\cos x = \dfrac{1}{4}$ and $\tan y = 6$.

Evaluate each of the following. (Your answers should be exact and supported by your work. Blank space is provided so that you can create a diagram.)





$\cos y = \frac{1}{\sqrt{37}}$

$\sin y = \frac{6}{\sqrt{37}}$

$\tan y = 6$

$\cos x = \frac{1}{4}$

$\sin x = \frac{\sqrt{15}}{4}$

$\tan x = \sqrt{15}$

(a) $\cos(x-y) = \cos x \cos y + \sin x \sin y = \frac{1}{4}\frac{1}{\sqrt{37}} + \frac{\sqrt{15}}{4}\frac{6}{\sqrt{37}}$

$= \dfrac{1+6\sqrt{15}}{4\sqrt{37}}$

(b) $\sin(x+y) = \sin x \cos y + \sin y \cos x$

$= \frac{\sqrt{15}}{4}\frac{1}{\sqrt{37}} + \frac{6}{\sqrt{37}}\frac{1}{4} = \dfrac{\sqrt{15}+6}{4\sqrt{37}}$

(c) $\sin\left(\frac{x}{2}\right) = \sqrt{\dfrac{1-\cos x}{2}} = \sqrt{\dfrac{1-1/4}{2}} = \sqrt{\dfrac{4-1}{8}} = \sqrt{\dfrac{3}{8}}$

$\frac{x}{2}$ is acute

(d) $\tan(x+y) = \dfrac{\tan x + \tan y}{1-\tan x \tan y} = \dfrac{\sqrt{15}+6}{1-6\sqrt{15}}$

6. (15 points) Find **all of the solutions** of the trigonometric equation.

$2\sin x - 1 = 0 \quad \Rightarrow \quad \sin x = \frac{1}{2}$

There are two solutions in $[0, 2\pi)$ in quads I and II.

$x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$

All possible solutions are given by

$x = \frac{\pi}{6} + 2\pi k$ or

$x = \frac{5\pi}{6} + 2\pi k$ for $k = 0, \pm 1, \pm 2, \ldots$

7. (15 points) Find all solutions in the interval $[0, 2\pi)$ of the trigonometric equation.

$2\cos^2\theta - \cos\theta - 1 = 0$ Factor $(2\cos\theta + 1)(\cos\theta - 1) = 0$

$2\cos\theta + 1 = 0$ or $\cos\theta - 1 = 0$

$\cos\theta = \frac{-1}{2}$ or $\cos\theta = 1$

The first has 2 solutions, one each in quadrants II and III,

$\theta = \frac{2\pi}{3}$ or $\theta = \frac{4\pi}{3}$

The second has one solution $\theta = 0$.

The solution set is $\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$.