Exam 4 Math 1113 sec. 51 Fall 2018

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ____________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

INSTRUCTIONS: There are 10 problems worth 10 points each. No calculator use is allowed on any part of this exam. There are no notes, or books allowed. **Illicit use of a calculator, smart phone, tablet, device that runs apps, or notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, and written using proper notation.

You may assume the following six IDs. Choose the signs with same placements.

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\sin(u \pm v) = \sin u \cos v \pm \sin v \cos u$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

1. Complete the table of trigonometric values.

$\theta =$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

2. Evaluate each expression exactly.

(a) $\cos\left(\frac{4\pi}{3}\right) = \frac{-1}{2}$

$\theta' = \frac{\pi}{3}$ quad III

(b) $\tan\left(\frac{5\pi}{4}\right) = 1$

$\theta' = \pi/4$ quad III

(c) $\csc\left(\frac{5\pi}{6}\right) = 2$

$\theta' = \frac{\pi}{6}$ quad II

(d) $\cot\left(-\frac{\pi}{3}\right) = \frac{-1}{\sqrt{3}}$

$\theta' = \frac{\pi}{3}$ quad IV

(e) $\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$

3. Complete the table. Use interval notation.

Function	Domain	Range
$y = \sin^{-1} x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$

4. Evaluate each expression exactly. Recall that certain formulas have been provided on the first page of this exam.

(a) $\sin(65°)\cos(25°) + \sin(65°)\cos(25°) = \sin(65° + 25°) = \sin(90°) = 1$

(b) $\cos(50°)\cos(5°) + \sin(50°)\sin(5°) = \cos(50° - 5°) = \cos(45°) = \frac{1}{\sqrt{2}}$

(c) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

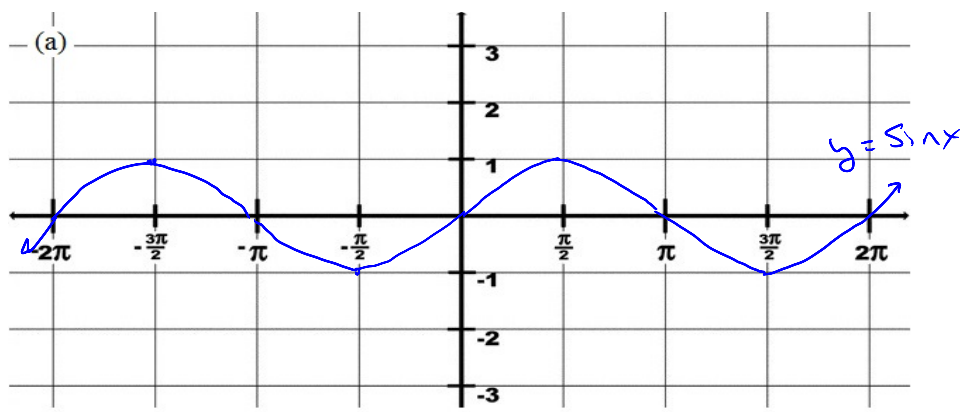
(d) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

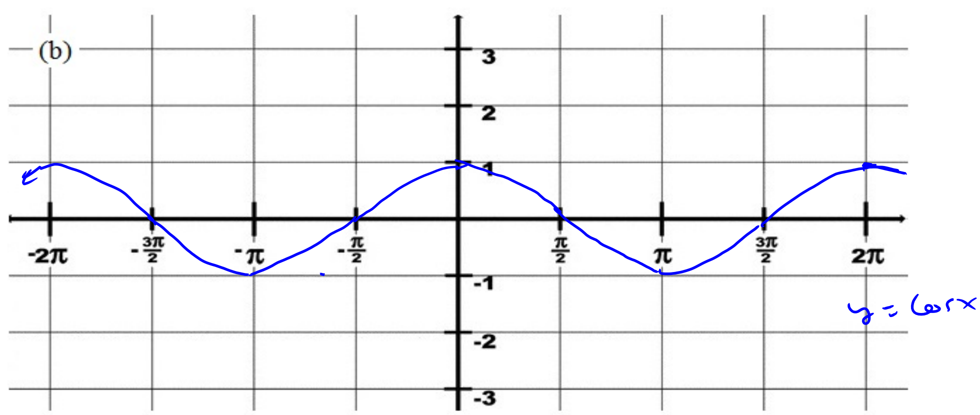
(e) $\tan\left(\frac{\pi}{12}\right)$ (hint: $\frac{1}{12} = \frac{1}{3} - \frac{1}{4}$)

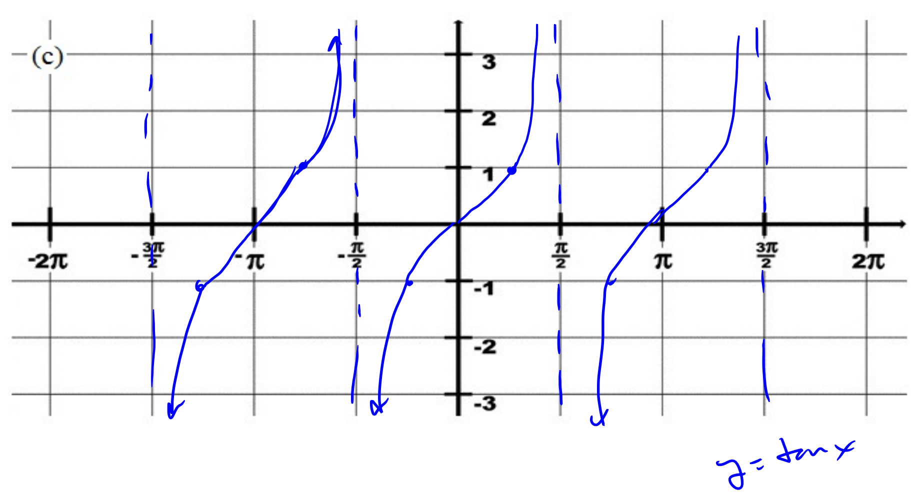
$$= \frac{\tan\left(\frac{\pi}{3}\right) - \tan\left(\pi/4\right)}{1 + \tan\frac{\pi}{3} \cdot \tan \pi/4} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$$

5. Plot at least two full periods of each function on the grids provided.

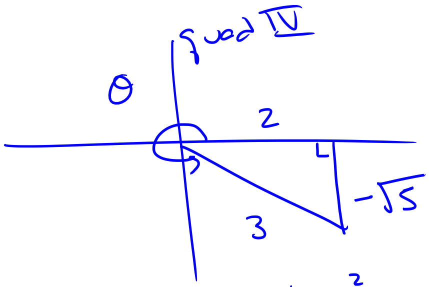
(a) $f(x) = \sin x$ (b) $g(x) = \cos x$ (c) $h(x) = \tan x$





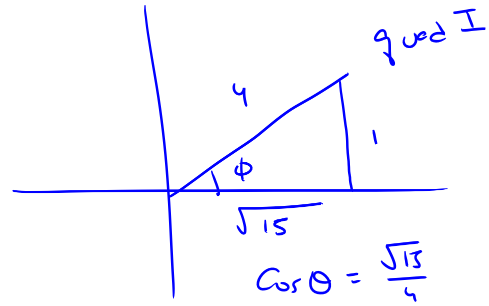


6. Suppose $\frac{3\pi}{2} < \theta < 2\pi$, and $\sec\theta = \frac{3}{2}$. Also suppose that $0 < \phi < \frac{\pi}{2}$ and $\sin\phi = \frac{1}{4}$. Draw a representative diagram for each angle from which trigonometric values can be deduced.



$\cos\theta = \frac{2}{3}$

$\sin\theta = \frac{-\sqrt{5}}{3}$



$\cos\theta = \frac{\sqrt{15}}{4}$

$\sin\theta = \frac{1}{4}$

Evaluate each expression exactly. Your answers should be simplified, but it is not necessary to rationalize denominators.

(a) $\cos(\phi - \theta)$ $= \cos\phi\cos\theta + \sin\phi\sin\theta = \frac{\sqrt{15}}{4}\cdot\frac{2}{3} + \frac{1}{4}\left(\frac{-\sqrt{5}}{3}\right)$

$= \frac{2\sqrt{15} - \sqrt{5}}{12}$

(b) $\csc(\theta + \phi)$ $= \frac{12}{2 - \sqrt{75}}$

$\sin(\theta+\phi) = \sin\theta\cos\phi + \sin\phi\cos\theta = \frac{-\sqrt{5}}{3}\left(\frac{\sqrt{15}}{4}\right) + \frac{1}{4}\left(\frac{2}{3}\right)$

$= \frac{2 - \sqrt{75}}{12}$

7. For each function, identify the amplitude A, period T, phase shift ϕ (with direction), and vertical shift V (with direction). Write *none* if a function does not have a specific characteristic.

(a) $y = 2 - \frac{1}{3}\sin\left(x - \frac{\pi}{4}\right)$

$A =$ $\frac{1}{3}$, $T =$ 2π, $\phi =$ $\pi/4$ right $V =$ 2 up

(b) $f(x) = 4\cos\left(\frac{\pi x}{4}\right) - 2$

$A =$ 4, $T =$ 8, $\phi =$ none $V =$ 2 down

$\frac{2\pi}{\pi/4} = 8$

8. The hour hand on a certain clock is 5 inches long. (Provide exact answers with the factor π if necessary.)

(a) Determine the distance traversed by the tip of the hour hand over the course of 7 hours. (in inches)

The distance $s = r\theta$ $\quad r = 5$ in $\quad \theta = \frac{7}{12}(2\pi) = \frac{7\pi}{6}$

so

$$s = (5 \text{ in})\left(\frac{7\pi}{6}\right) = \frac{35\pi}{6} \text{ in}$$

(b) Find the area of the sector swept out by the hour hand during this 7 hours. (in square inches)

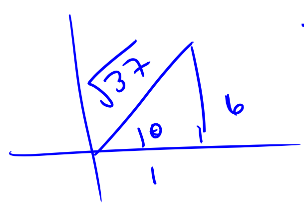
The area $A = \frac{1}{2}r^2\theta$. Using r and θ

$$A = \frac{1}{2}(5 \text{ in})^2\left(\frac{7\pi}{6}\right) = \frac{175\pi}{12} \text{ in}^2$$

9. Evaluate each expression exactly. Your answers should be justified by a diagram or other demonstration of your process.

(a) $\sin\left(\tan^{-1} 6\right)$

Let $\theta = \tan^{-1} 6$



$\tan\theta = 6$

$0 < \theta < \pi/2$

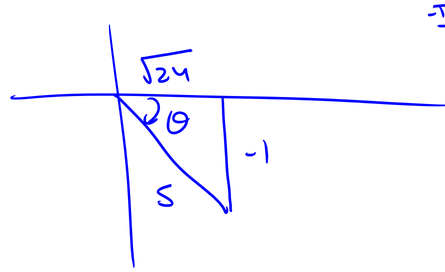
$= \dfrac{6}{\sqrt{37}}$ from the diagram

Let $\theta = \sin^{-1}\left(\frac{-1}{5}\right)$

$-\frac{\pi}{2} < \theta < 0$

$\sin\theta = \frac{-1}{5}$

(b) $\cot\left(\sin^{-1} -\dfrac{1}{5}\right) = -\sqrt{24}$



10. Prove each identity.

(a) $\sec^4 x - \tan^4 x = \sec^2 x + \tan^2 x$

From the left

$$\sec^4 x - \tan^4 x = (\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$$

$$= 1 \cdot (\sec^2 x + \tan^2 x)$$

$$= \sec^2 x + \tan^2 x \qquad \text{as required}$$

(b) $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta} = 2\sec^2\theta$

From the left

$$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = \frac{1-\sin\theta + 1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta}$$

$$= \frac{2}{\cos^2\theta}$$

$$= 2\sec^2\theta \qquad \text{as expected}$$