Exam 4 Math 1190 sec. 51

Fall 2016

Name: (2 points) Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

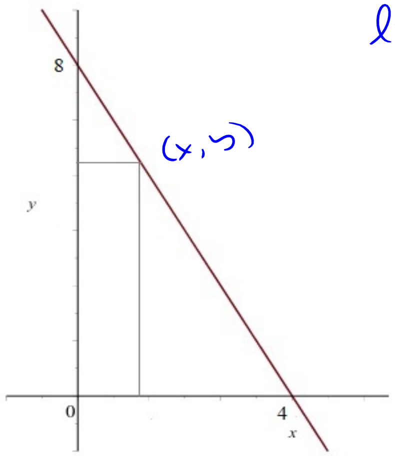
Problem	Points
1	
2	
3	
4	
5	
6	
7	

INSTRUCTIONS: There are 7 problems worth 14 points each.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.**

To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) Find the area of the largest rectangle that can be inscribed in the first quadrant below the line with intercepts $(0, 8)$ and $(4, 0)$ (see the figure).



line: $m = \frac{8-0}{0-4} = -2$ y-inc. is 8

$$y = -2x + 8$$

The area of a rectangle is

$A = xy$ when $y = -2x + 8$

$$A = x(-2x+8) = -2x^2 + 8x$$

$A'(x) = -4x + 8$ $A'(x) = 0 \Rightarrow -4x + 8 = 0 \Rightarrow x = 2$

$A''(x) = -4$ so $A''(2) = -4 < 0$ concave down

we have a maximizer by the 2nd der. test

The maximum area is

$$A(2) = -2 \cdot 2^2 + 8 \cdot 2 = -8 + 16 = 8$$

(2) Consider the function $\quad g(x) = \int_1^{x^2} e^{\sqrt{t}}\,dt$

(a) Evaluate $g(1)$.

$g(1) = \int_1^{1^2} e^{\sqrt{t}}\,dt = \int_1^{1} e^{\sqrt{t}}\,dt = 0$

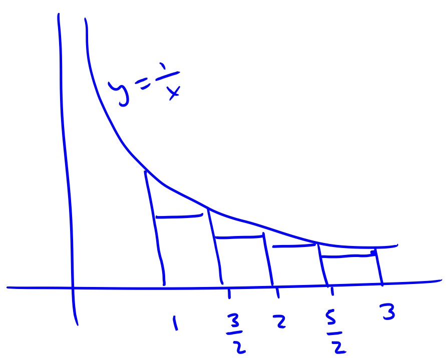
(b) Find the derivative $g'(x)$.

$g'(x) = e^{\sqrt{x^2}} \cdot (2x) = 2x\,e^{x}$

(3) Approximate the area under the curve $y = \frac{1}{x}$ above the x-axis and between the vertical lines $x = 1$ and $x = 3$ using four rectangles of equal width and right end points. You may leave your answer as a sum of numbers (no need to do the arithmetic, but there should be no variables in your final answer).

$x_0 = 1 \quad x_1 = \frac{3}{2}, \quad x_2 = 2, \quad x_3 = \frac{5}{2}, \quad x_4 = 3$

$\Delta x = \frac{3-1}{4} = \frac{1}{2}$



$f(x_1) = \frac{1}{3/2} = \frac{2}{3} \qquad f(x_3) = \frac{1}{5/2} = 2/5$

$f(x_2) = \frac{1}{2} \qquad f(x_4) = \frac{1}{3}$

$A \approx f(x_1)\,\Delta x + f(x_2)\,\Delta x + f(x_3)\,\Delta x + f(x_4)\,\Delta x$

$= \frac{2}{3}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} + \frac{2}{5}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2}$

(4) Evaluate each definite integral using the Fundamental Theorem of Calculus.

(a) $\int_0^2 (6x^2 - 1)\, dx$ $= 6\frac{x^3}{3} - x \Big|_0^2 = 2x^3 - x \Big|_0^2$

$= 2(2)^3 - 2 - (2 \cdot 0^3 - 0) = 16 - 2 = 14$

(b) $\int_1^2 \frac{xe^x + 1}{x}\, dx$ $= \int_1^2 (e^x + \frac{1}{x})\, dx = e^x + \ln|x| \Big|_1^2$

$= e^2 + \ln|2| - (e^1 + \ln|1|) = e^2 + \ln 2 - e$

(5) Solve the differential equation subject to the given condition[1].

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}} \quad \text{when} \quad x = \frac{1}{2} \quad y = 0$$

$y = 2\sin^{-1}x + C$

$y(\frac{1}{2}) = 2\sin^{-1}(\frac{1}{2}) + C = 0$

$2(\frac{\pi}{6}) + C = 0 \Rightarrow C = \frac{-2\pi}{6} = \frac{-\pi}{3}$

$\boxed{y = 2\sin^{-1}x - \frac{\pi}{3}}$

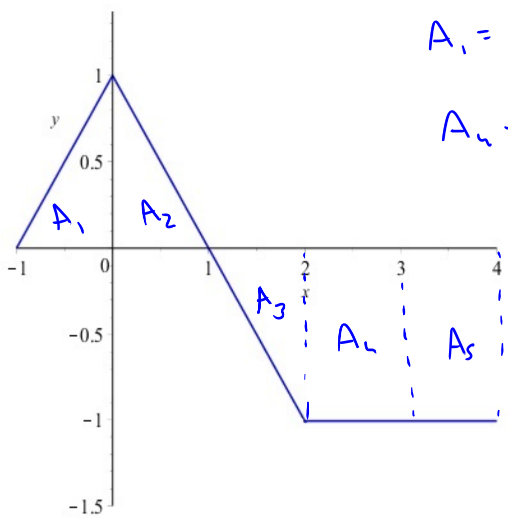
[1]The following may be useful to remember $\sin\frac{\pi}{6} = \frac{1}{2}$.

(6) Find the average value of the function $f(x) = x^3$ over the interval $[0, 2]$.

$$f_{avg} = \frac{1}{2-0} \int_0^2 x^3 \, dx = \frac{1}{2} \left[\frac{x^4}{4} \right|_0^2$$

$$= \frac{1}{2} \left[\frac{2^4}{4} - \frac{0^4}{4} \right] = \frac{1}{2} \left(\frac{16}{4} \right) = \frac{4}{2} = 2$$

(7) The graph of $y = f(x)$ is shown in the figure. Use the figure to evaluate the integrals.



$A_1 = A_2 = A_3 = \frac{1}{2}(1)(1) = \frac{1}{2}$

$A_4 = A_5 = 1 \cdot 1 = 1$

(a) $\displaystyle\int_{-1}^{3} f(x)\, dx = A_1 + A_2 - A_3 - A_4 = \frac{1}{2} + \frac{1}{2} - \frac{1}{2} - 1 = \frac{-1}{2}$

(b) $\displaystyle\int_{0}^{4} f(x)\, dx = A_2 - A_3 - A_4 - A_5 = \frac{1}{2} - \frac{1}{2} - 1 - 1 = -2$