Exam 4 Math 1190 sec. 51

Summer 2017

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	

INSTRUCTIONS: You have 60 minutes to complete this exam.

There are 7 problems. The point values are listed with the problems.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, justified, and written using proper notation.

. . . .

The following may or may not be useful:

$$\sin 0 = 0, \quad \cos 0 = 1, \quad \tan 0 = 0$$

$$\sin\frac{\pi}{6} = \frac{1}{2}, \quad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}, \quad \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}, \quad \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}, \quad \tan\frac{\pi}{4} = 1$$

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}, \quad \cos\frac{\pi}{3} = \frac{1}{2}, \quad \tan\frac{\pi}{3} = \sqrt{3}$$

(1) (25 points, 5 each) Evaluate each definite integral. (Do not leave compound fractions in any of your answers.)

(a) $\displaystyle\int_1^e \frac{1}{x}\,dx = \ln|x| \Big|_1^e = \ln|e| - \ln|1| = 1 - 0 = 1$

(b) $\displaystyle\int_1^4 \frac{3}{2}x^{1/2}\,dx = x^{3/2} \Big|_1^4 = 4^{3/2} - 1^{3/2} = 8 - 1 = 7$

(c) $\displaystyle\int_0^{\frac{\pi}{2}} \sin x\,dx = -\cos x \Big|_0^{\pi/2} = -\cos \pi/2 - (-\cos 0)$

$= 0 - (-1) = 1$

(d) $\displaystyle\int_1^2 e^x\,dx = e^x \Big|_1^2 = e^2 - e^1 = e^2 - e$

(e) $\displaystyle\int_0^{\frac{\pi}{3}} \sec x \tan x\,dx = \sec x \Big|_0^{\pi/3} = \sec \pi/3 - \sec 0 = 2 - 1 = 1$

(2) (20 points, 5 each) Evaluate each definite integral. Simplify to the extent possible.

(a) $\displaystyle\int_1^2 3x(x+2)\,dx = \int_1^2 (3x^2+6x)\,dx$

$= x^3+3x^2\Big|_1^2 = 2^3+3(2^2)-(1^3+3(1)^2) = 8+12-4$

$= 16$

(b) $\displaystyle\int_0^{\frac{\pi}{4}} (\sqrt{2}\,\cos\theta+\sec^2\theta)\,d\theta = \sqrt{2}\,\sin\theta+\tan\theta\Big|_0^{\pi/4}$

$= \sqrt{2}\,\sin\frac{\pi}{4}+\tan\frac{\pi}{4}-(\sqrt{2}\sin 0+\tan 0)$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)+1-0 = 2$

(c) $\displaystyle\int_1^3 \frac{4t^2-3t+2}{t}\,dt = \int_1^3\left(4t-3+\frac{2}{t}\right)dt = 2t^2-3t+2\ln|t|\Big|_1^3$

$= 2(3^2)-3(3)+2\ln|3|-(2(1^2)-3\cdot 1+2\ln 1)$

$= 18-9+2\ln 3-(-1+0)$

$= 10+2\ln 3$

(d) $\displaystyle\int_1^{\sqrt{3}} \frac{12}{1+x^2}\,dx = 12\tan^{-1}x\Big|_1^{\sqrt{3}} = 12\tan^{-1}\sqrt{3}-12\tan^{-1}1$

$= 12\left(\frac{\pi}{3}\right)-12\left(\frac{\pi}{4}\right) = 4\pi-3\pi = \pi$

(3) (10 points) A spherical balloon is being inflated. Air is being pumped in at a constant rate of 20 cm^3 per second. Find the rate at which the surface area is increasing at the moment that the radius is 10 cm. (Include appropriate units.)

$V = \frac{4}{3}\pi r^3, \quad S = 4\pi r^2 \qquad \frac{dV}{dt} = 20 \frac{\text{cm}^3}{\text{sec}}$

find $\frac{dS}{dt}$ when $r = 10$ cm.

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$ and $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2} \frac{dV}{dt}$

So $\frac{dS}{dt} = 8\pi r \left(\frac{1}{4\pi r^2}\right)\frac{dV}{dt} = \frac{2}{r}\frac{dV}{dt}$

when $r = 10$ cm

$\frac{dS}{dt} = \frac{2}{10\text{cm}}\left(20 \frac{\text{cm}^3}{\text{sec}}\right) = 4 \frac{\text{cm}^2}{\text{sec}}$

The surface area is increasing at a rate of 4 cm^2 per second.

The following may or may not be useful:

- The volume of a sphere of radius r is $\frac{4\pi}{3}r^3$
- The balloon is orange.
- The volume of a cylinder of radius r and height h is $\frac{\pi}{3}r^2h$
- The surface area of a sphere of radius r is $4\pi r^2$
- Orange rhymes with *Blorenge*.

(4) (10 points) Set up the Newton's Method iteration formula that could be used to find the positive real root of the polynomial $f(x) = x^4 - x^3 + 2x - 4$. Starting with an initial guess of $x_0 = 1$, compute the next iterate x_1.

$f'(x) = 4x^3 - 3x^2 + 2$

$f(1) = 1 - 1 + 2 - 4 = -2$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f'(1) = 4 - 3 + 2 = 3$

$$\boxed{x_{n+1} = x_n - \frac{x_n^4 - x_n^3 + 2x_n - 4}{4x_n^3 - 3x_n^2 + 2}}$$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{-2}{3} = 1 + \frac{2}{3} = \frac{5}{3}$

$x_0 = 1, \quad x_1 = \frac{5}{3}$

(5) (10 points) Find the average value of the function $f(x) = \dfrac{1}{\sqrt{x}}$ over the interval $[1, 9]$.

$f_{avg} = \frac{1}{9-1} \int_1^9 x^{-1/2}\, dx$

$= \frac{1}{8} \left[\frac{x^{1/2}}{1/2} \right|_1^9 = \frac{1}{8} \left(2\sqrt{x} \right|_1^9$

$= \frac{1}{8} \left(2\sqrt{9} - 2\sqrt{1} \right) = \frac{1}{8} (6 - 2)$

$= \frac{4}{8} = \frac{1}{2}$

(6) (10 points) Find the area of the largest rectangle that can be inscribed in the first quadrant under the parabola $y = 27 - x^2$.

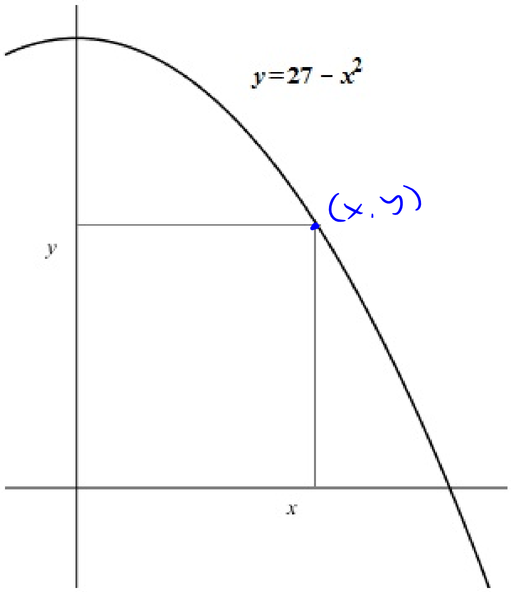

Figure 1: Example of an inscribed rectangle.

If the rectangle hits the curve @ (x, y), the area is

$$A = xy = x(27 - x^2)$$

Find crit. #:

$$A'(x) = 27 - 3x^2 = 3(9 - x^2)$$

$A'(x)$ is always defined

$A'(x) = 0 \Rightarrow x = \pm 3$

Since $x > 0$, $x = 3$ is the only crit. # of interest.

2nd der. test $\quad A''(x) = -6x \quad A''(3) = -18 < 0$

we have a max.

The maximal area is

$$A(3) = 3(27 - 3^2) = 3(18) = 54$$

(7) (15 points, 5 each) Evaluate each derivative.

(a) $\frac{d}{dx}\left[\int_1^x t\ln t\,dt\right]$ $= x\ln x$

(b) $\frac{d}{dx}\left[\int_x^{10} e^t \sin t\,dt\right]$ $= -e^{x} \sin x$

(c) $\frac{d}{dx}\left[\int_x^{2x} \cos^3 t\,dt\right]$ $= 2\cos^3(2x) - \cos^3(x)$