Exam 4 Math 1190 sec. 62

Spring 2017

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	

INSTRUCTIONS: You have 50 minutes to complete this exam.

There are 7 problems. The point values are listed with the problems.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) Use the Fundamental Theorem to evaluate each definite integral[1]. Simplify your answers as much as possible.

(a) (10 points) $\displaystyle\int_1^3 \frac{2x^2+1}{x}\,dx \quad = \int_1^3 \left(2x+\frac{1}{x}\right)dx$

$$= x^2 + \ln|x| \Big|_1^3 = 3^2 + \ln 3 - (1^2 + \ln 1)$$

$$= 9 + \ln 3 - 1 - 0 = 8 + \ln 3$$

(b) (10 points) $\displaystyle\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-y^2}}\,dy \quad = \sin^{-1} y \Big|_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}}$

$$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{4} - \frac{\pi}{6} = \frac{3\pi}{12} - \frac{2\pi}{12} = \frac{\pi}{12}$$

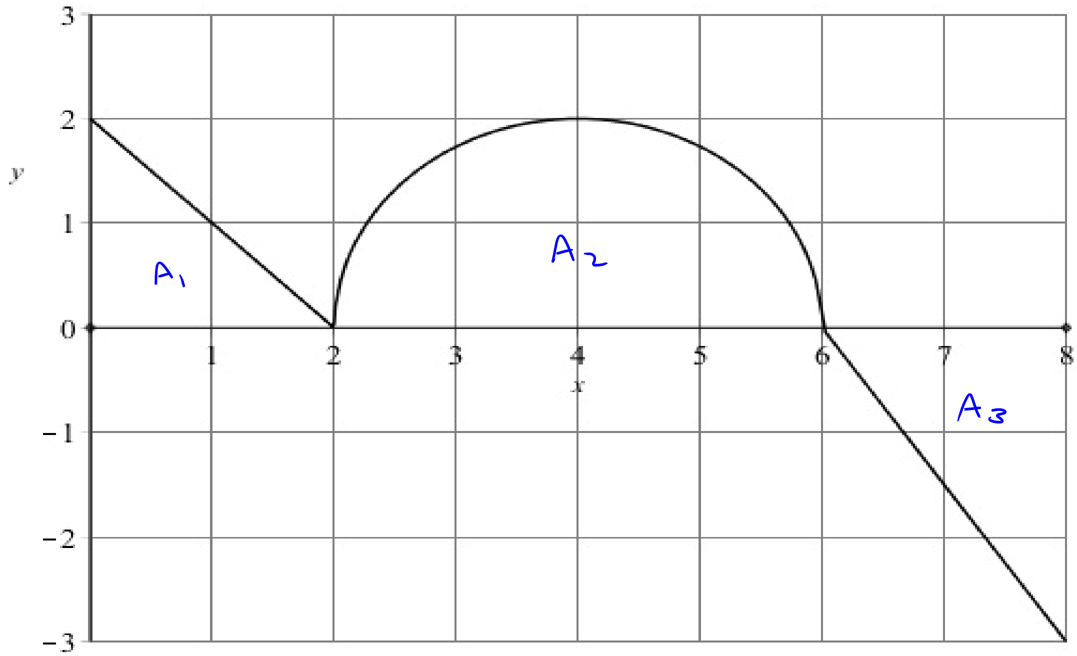
[1]The following may or may not be useful

$$\sin 0 = 0, \quad \cos 0 = 1, \quad \tan 0 = 0,$$

$$\sin\frac{\pi}{6} = \frac{1}{2}, \quad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}, \quad \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}},$$

$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}, \quad \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}, \quad \tan\frac{\pi}{4} = 1.$$

(2) The graph of $y = f(x)$ is shown. The curve on the interval $2 \leq x \leq 6$ is a semicircle. Use the



$A_1 = \frac{1}{2}(2)(2) = 2$

$A_2 = \frac{1}{2}\pi(2^2) = 2\pi$

$A_3 = \frac{1}{2}(2)(3) = 3$

figure to evaluate each integral. (Your answers should be exact values, not approximations.)

(a) (6 points) $\displaystyle\int_0^6 f(x)\,dx$ $= 2 + 2\pi$ $\quad A_1 + A_2$

(b) (6 points) $\displaystyle\int_2^8 f(x)\,dx$ $= 2\pi - 3$ $\quad A_2 - A_3$

(3) Evaluate each definite integral.

(a) (10 points) $\displaystyle\int_0^2 x(3x+1)\,dx$ $= \displaystyle\int_0^2 (3x^2 + x)\,dx = x^3 + \frac{x^2}{2}\Big|_0^2 = 2^3 + \frac{2^2}{2} - \left(0^3 + \frac{0^2}{2}\right)$

$= 8 + 2 - 0 = 10$

(b) (10 points) $\displaystyle\int_{-1}^0 2e^{2x}\,dx$

$= e^{2x}\Big|_{-1}^{0} = e^0 - e^{-2} = 1 - e^{-2}$

(4) Evaluate each indicated derivative.

(a) (10 points) $\dfrac{d}{dx}\displaystyle\int_3^x \cos(2^t)\,dt \quad = \quad \cos(2^x)$

(b) (10 points) $\dfrac{d}{dx}\displaystyle\int_1^{\frac{1}{x}} t\ln(t)\,dt \quad = \frac{1}{x}\ln\left(\frac{1}{x}\right)\cdot\left(\frac{-1}{x^2}\right)$ $\qquad \frac{d}{dx}x^{-1} = -x^{-2} = \frac{-1}{x^2}$

$= \dfrac{-\ln\frac{1}{x}}{x^3} = \dfrac{\ln x}{x^3}$ $\qquad * -\ln\frac{1}{x} = \ln x$

(5) (8 points) A spherical snow ball grows as it rolls down a hill. At the moment that the volume[2] of the snow ball is $\frac{4}{3}\pi$ cubic feet, the volume is increasing at a rate of $\frac{1}{10}^{th}$ cubic feet per second. Determine the rate at which the radius of the snow ball is changing when its volume is $\frac{4}{3}\pi$ cubic feet. (Include units in your answer.)

$V = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad \frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}$

when $V = \frac{4}{3}\pi\ ft^3$, $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \Rightarrow r^3 = 1 \Rightarrow r = 1\ ft$

At this moment

$$\frac{1}{10}\frac{ft^3}{sec} = 4\pi(1ft)^2\cdot\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{10}\frac{ft^3}{sec}\frac{1}{4\pi(1^2 ft^2)} = \frac{1}{40\pi}\ ft/sec$$

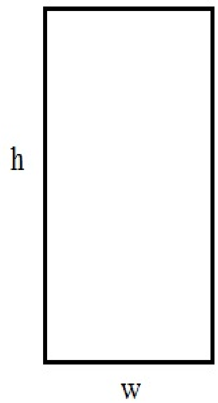
The radius is increasing at a rate of $\frac{1}{40\pi}$ ft per sec.

---

[2]The volume V and surface area S of a sphere with radius r are

$$V = \frac{4}{3}\pi r^3 \quad \text{and} \quad S = 4\pi r^2.$$

(6) (8 points) The rectangle shown has height h and width w. Suppose that when $h = 4$ meters, and $w = 2$ meters, the height is decreasing at a rate of 50 cm/min (i.e. $\frac{1}{2}$ m/min) and the width is increasing at a rate of 75 cm/min (i.e. $\frac{3}{4}$ m/min). Determine the rate at which the area is changing when the height and width are 4 and 2 meters, respectively. (Include appropriate units in your answer.)



$A = hw$ when $h = 4$ m, $w = 2$ m

$\frac{dh}{dt} = -\frac{1}{2}\ \frac{m}{min}$

$\frac{dw}{dt} = \frac{3}{4}\ \frac{m}{min}$

$\frac{dA}{dt} = h\frac{dw}{dt} + w\frac{dh}{dt}$

So when $h=4$ and $w=2$

$\frac{dA}{dt} = 4\,m\left(\frac{3}{4}\frac{m}{min}\right) + 2m\left(\frac{-1}{2}\frac{m}{min}\right)$

$= 3\ \frac{m^2}{min} - 1\ m^2/min = 2\ \frac{m^2}{min}$

Area is increasing at a rate of $2\ \frac{m^2}{min}$.

(7) (a) (6 points) Show that $\frac{d}{dx}(x \ln x - x) = \ln x$.

$\frac{d}{dx}(x\ln x - x) = 1 \cdot \ln x + x \cdot \frac{1}{x} - 1 = \ln x + 1 - 1 = \ln x$

(b) (6 points) Use the above result to evaluate the integral

$\int_1^e \ln x\, dx = x\ln x - x \Big|_1^e = e\ln e - e - (1\ln 1 - 1)$

$= e \cdot 1 - e - (0 - 1)$

$= 0 - (-1) = 1$