Exam 4 Math 1190 sec. 63

Spring 2017

Name: Solutions

Your signature (required) confirms that you agree to practice academic honesty.

Signature: ______________________________

Problem	Points
1	
2	
3	
4	
5	
6	
7	

INSTRUCTIONS: You have 50 minutes to complete this exam.

There are 7 problems. The point values are listed with the problems.

There are no notes, or books allowed and **no calculator is allowed. Illicit use of a calculator, smart phone, tablet, device that runs apps, or hand written notes will result in a grade of zero on this exam as well as a formal allegation of academic misconduct.** To receive full credit, answers must be clear, complete, justified, and written using proper notation.

(1) Use the Fundamental Theorem to evaluate each definite integral[1]. Simplify your answers as much as possible.

(a) (10 points) $\displaystyle\int_1^3 \frac{1-2x^2}{x}\,dx$ $= \displaystyle\int_1^3 \left(\frac{1}{x} - 2x\right) dx$

$= \ln|x| - x^2 \Big|_1^3 = \ln|3| - 3^2 - (\ln|1| - 1^2)$

$= \ln 3 - 9 - (0-1)$

$= \ln 3 - 9 + 1 = \ln 3 - 8$

(b) (10 points) $\displaystyle\int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-z^2}}\,dz$ $= \sin^{-1} z \Big|_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}}$

$= \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{\sqrt{2}}$

$= \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

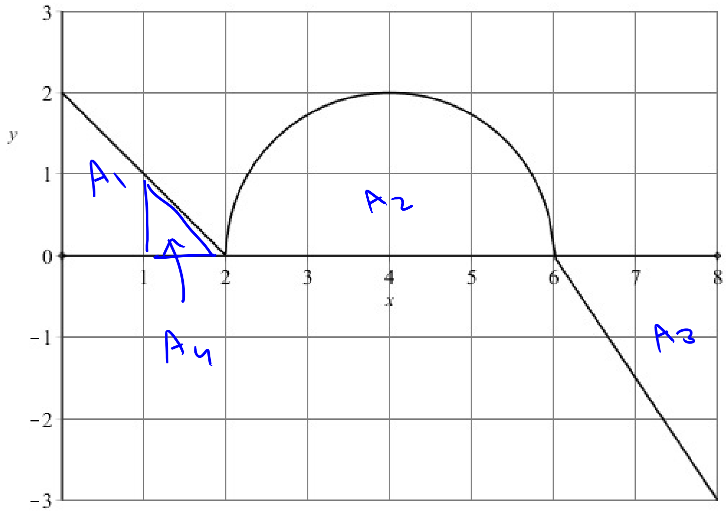
[1]The following may or may not be useful

$$\sin 0 = 0, \quad \cos 0 = 1, \quad \tan 0 = 0,$$

$$\sin\frac{\pi}{6} = \frac{1}{2}, \quad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}, \quad \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}},$$

$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}, \quad \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}, \quad \tan\frac{\pi}{4} = 1.$$

(2) The graph of $y = f(x)$ is shown. The curve on the interval $2 \le x \le 6$ is a semicircle. Use the



$A_1 = \frac{1}{2}(2)(2) = 2$

$A_2 = \frac{1}{2}\pi(2)^2 = 2\pi$

$A_3 = \frac{1}{2}(2)(3) = 3$

$A_4 = \frac{1}{2}(1)(1) = \frac{1}{2}$

figure to evaluate each integral. (Your answers should be exact values, not approximations.)

(a) (6 points) $\displaystyle\int_0^8 f(x)\,dx = 2 + 2\pi - 3 = 2\pi - 1$ $\qquad A_1 + A_2 - A_3$

(b) (6 points) $\displaystyle\int_1^6 f(x)\,dx = \frac{1}{2} + 2\pi$ $\qquad A_4 + A_2$

(3) Evaluate each definite integral using any applicable technique.

(a) (10 points) $\displaystyle\int_1^2 x(3x+4)\,dx = \int_1^2 (3x^2+4x)\,dx = x^3 + 2x^2 \Big|_1^2$

$= 2^3 + 2\cdot 2^2 - (1^3 + 2\cdot 1^2) = 8 + 8 - 3 = 13$

(b) (10 points) $\displaystyle\int_{-1}^1 3e^{3x}\,dx = e^{3x}\Big|_{-1}^{1} = e^3 - e^{-3}$

(4) Evaluate each indicated derivative.

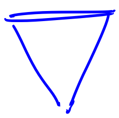
(a) (10 points) $\dfrac{d}{dx}\displaystyle\int_2^x \cot(t^3)\,dt$ $= \cot(x^3)$

(b) (10 points) $\dfrac{d}{dx}\displaystyle\int_1^{\sqrt{x}} t\ln(t)\,dt$ $= \sqrt{x}\,\ln(\sqrt{x}) \cdot \dfrac{1}{2\sqrt{x}}$

$\dfrac{d}{dx} x^{1/2} = \dfrac{1}{2} x^{-1/2} = \dfrac{1}{2\sqrt{x}}$

$= \dfrac{1}{2}\ln\sqrt{x}$

(5) (8 points) Sand is dumped into a pile in the shape of a cone such that the height is always twice the base radius[2]. At the moment that the volume of the pile is $\frac{2\pi}{3}$ cubic meters, the volume is increasing at a rate of $\frac{1}{2}$ cubic meters per minute. Determine the rate at which the base radius is changing when the volume of the pile is $\frac{2\pi}{3}$ cubic meters. (Include units in your answer.)



$V = \frac{2\pi}{3} r^3$. When $V = \frac{2\pi}{3}\ m^3$, $\frac{2\pi}{3} = \frac{2\pi}{3} r^3$

$\Rightarrow r = 1\ m$

Given $\frac{dV}{dt} = \frac{1}{2}\ m^3/min$

when $r = 1\ m$

$$\frac{dV}{dt} = \frac{2\pi}{3}(3r^2)\frac{dr}{dt} = 2\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{2\pi r^2}\frac{dV}{dt}$$

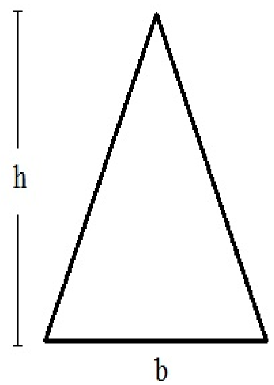
So when $r = 1\ m$,

$$\frac{dr}{dt} = \frac{1}{2\pi(1m)^2} \cdot \frac{1}{2}\frac{m^3}{min} = \frac{1}{4\pi}\frac{m}{min}$$

The radius is increasing at a rate of $\frac{1}{4\pi}$ meters per minute at this moment.

[2]The volume V of a cone of base radius r and height $2r$ is $V = \frac{2\pi}{3}r^3$

(6) (8 points) The triangle shown has height h and base length b. Suppose that when $h = 6$ meters, and $b = 4$ meters, the height is decreasing at a rate of 1 m/hr and the base is increasing at a rate of 2 m/hr. Determine the rate at which the area is changing when the height and base are 6 and 4 meters, respectively. (Include appropriate units in your answer.)



$A = \frac{1}{2}bh$ Given, when $h = 6m$ and $b = 4m$

$\frac{dh}{dt} = -1 \frac{m}{hr}$ and $\frac{db}{dt} = 2\frac{m}{hr}$

$\frac{dA}{dt} = \frac{1}{2}b\frac{dh}{dt} + \frac{1}{2}h\frac{db}{dt}$

So when $h=6$ and $b=4$

$\frac{dA}{dt} = \frac{1}{2}4m\left(-1\frac{m}{hr}\right) + \frac{1}{2}\cdot 6m\left(2\frac{m}{hr}\right)$

$= -2\ m^2/hr + 6m^2/hr = 4\ m^2/hr$

At this time, the area is increasing at a rate of 4 m^2 per hour.

(7) (a) (6 points) Show that $\frac{d}{dx}(\sin x - x\cos x) = x \sin x$.

$\frac{d}{dx}(\sin x - x\cos x) = \cos x - 1\cdot\cos x - x(-\sin x)$

$= \cos x - \cos x + x\sin x = x\sin x$

(b) (6 points) Use the above result to evaluate the integral

$\int_0^{\frac{\pi}{2}} x \sin x\, dx = \sin x - x\cos x \Big|_0^{\pi/2} = \sin\frac{\pi}{2} - \frac{\pi}{2}\cos\frac{\pi}{2} - (\sin 0 - 0\cos 0)$

$= 1 - 0 - (0-0) = 1$