Perspective and Passion in Art, Mathematics and Pedagogy

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UTBM
Why was mathematics 300 years behind art in the study of perspective and projective geometry?
Late 12th century Macedonia
Giotto, *Lamentation Over The Dead Christ* 1305
Pentacost, The Arena Chapel
Rules of Perspective

(1) The image of a straight line is a straight line

(2) The image of a conic section is a conic section

(3) The image of parallel lines is concurrent
Conic Sections

- parabola
- circle
- ellipse
- hyperbola
The image of a conic section is a conic section.
The image of a pair of parallel lines converges.
Lorenzetti, *Presentation of Christ in The Temple*, 1342
Leonardo's illustrations for Pacioli's *De divina proportione*, pub 1509
A cone in the drawing style used during most of the 15\textsuperscript{th} century
Different Agendas

Art  What an observer sees

Mathematics  Space as it actually is
Artists

Abacus Schools from late 13th c.
Brunelleschi (1377-1446) Artificial Perspective
Alberti (1404-1472) *Della Pittura* (1436)
Mathematicians

Greek Mathematics

Mathematics of the Islamic Period
From Euclid's *Optics*

[T]he figure enclosed by the sight-lines is a cone having its vertex at the eye and its base at the limits of the things seen...

[T]hings seen by a larger angle appear larger, while things seen by a smaller angle appear smaller.
Ibn al-haytham's theory of vision
Albrecht Durer's Perspective Machine
The same magnitude...viewed from near and from far does not appear equal...

[S]cene-painting in its exploitation of this weakness of our nature falls nothing short of witchcraft
Have not measuring and numbering and weighing proved to be most gracious aids to prevent the domination in our soul of the apparently greater or less or more or heavier?

The part of the soul...which puts its trust in measurement and reckoning must be the best part of the soul.

---Plato, *The Republic*
The Elements of Euclid

A codification of what was known about geometry at the time, around 300 BC
The earliest known axiomatic treatment of mathematics

The standard upon which school mathematics was based since the middle of the 19th century
Book I of *The Elements*

- 23 Definitions
- 5 Postulates (axioms)
- 5 Common Notions
- 48 Propositions (theorems)
Euclid's Postulates

1. A straight line can be drawn from any point to any point.
2. A finite straight line can be produced continuously in a straight line.
3. One may describe a circle with any center and any radius.
4. All right angles are equal to one another.
Euclid's Fifth Postulate

That, if a straight line falling on two straight lines make an interior angle on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.
Prominent Attackers

Euclid 300 BC
Proclus 450 AD
Ibn al-Haytham 1015
Omar Khayyam 1100
John Wallis 1656
Lagrange 1776
Legendre 1800
Gauss 1817
Equivalents to Postulate 5

1. Given a line and a point not on the line, there is exactly one parallel to the line through the point.
2. The angles of a triangle add up to two right angles.
3. There exist noncongruent similar triangles.
4. Alternate interior angles created by a transversal and two parallel lines are congruent.
“Detest it as lewd intercourse, it can deprive you of all your leisure, your health, your rest and the whole happiness of your life.”

Farkas Bolyai in a letter to his son János, responding to János's report that he was trying to prove the parallel postulate
Breakthroughs on the Parallel Postulate

Discoveries of Bolyai and Lobachevsky 1820s

Riemann's Habilitation 1854

Beltrami's interpretation 1868
Poincaré Disk Model
Projective Geometry as Mathematics

Poncelet's Treatise 1822

Plücker's homogeneous coordinates 1831

Felix Klein's algebraic foundation 1871
Projective Plane

(1) There exist four points no three of which are collinear
(2) Two points determine a unique line
(3) Two lines intersect in a unique point
Pappus's Theorem ~ 320 CE
Desargues' Theorem, 1648
What Happened Next?

Hilbert
Einstein
Bourbaki— a return to The Elements
Dalí, *Christ of Saint John of the Cross*, 1951
Brief List of References

Boyer, *A History of Mathematics*

Coolidge, *A History of Geometrical Methods*

Coxeter, *Projective Geometry*


Field, *The Invention of Infinity*

Grabiner, “Why did Lagrange `prove' the parallel postulate?” MAA Monthly, Jan 2009

Joyce, David, *Euclid's Elements*, interactive website


Struik, *A Concise History of Mathematics*