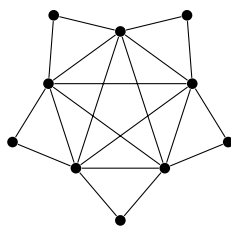# Graph Theory Homework 8

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due Friday, December 3, 2021

1   Short answer

1. Draw the octahedron graph in the plane without crossings, and find a vertex coloring of the octahedron graph with 3 colors.

2. Let G be the graph shown below.

   (a) Find the maximum degree $\Delta(G)$. What inequality on the chromatic number can we get from this maximum degree?

   (b) Find the clique number $\omega(G)$. What inequality on the chromatic number can we get from this clique number?

   (c) Find the independence number $\alpha(G)$. What inequality on the chromatic number can we get from this independence number?

   (d) Find the chromatic number $\chi(G)$.

3. In the *graph coloring game*, two players take turns coloring the vertices of a graph. There is a supply of available colors; a turn in the game consists of picking an uncolored vertex of the graph, and giving it a color different from any of the adjacent vertices.

   The first player wins if the graph is fully colored. The second player wins if it becomes impossible to finish the game: if there is an uncolored vertex which cannot be given a color.

   Suppose that this game is played on the cube graph, with two colors available: red and blue. How can the second player win?

2   Proof

4. Prove that an n-vertex graph G with maximum degree $\Delta(G)$ has an independent set with at least $\frac{n}{\Delta(G)+1}$ vertices.

   You have already written a rough draft of this solution; now, write a final draft.